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RIVER SURVEY PROJECT

Special Report No. 13

Sediment transport predictors

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Notation

Symbol	meaning	dimension
a	reference level above the mean bed	(m)
b	exponent of flow velocity	(-)
C	over-all Chezy coefficient	($m^{0.5}/s$)
C'	grain-related Chezy coefficient	($m^{0.5}/s$)
C_{90}	Chezy roughness coefficient based on d_{90}	($m^{0.5}/s$)
Cs	mean sediment concentration over the vertical	(mg/l)
c_a	reference concentration at level 'a'	(-)
c_i	concentration in point i above the bed	(mg/l)
c_t	total load concentration by weight	(ppm)
c_o	maximum concentration (= 0.65)	(-)
d	point depth	(m)
d	diameter of the sphere	(m)
d_s	representative particle size of suspended sediment	(m)
d_{35}	representative diameter of bed material	(m)
d_{50}	median particle diameter of bed material	(m)
d_{90}	characteristic diameter of bed material	(m)
D_*	dimensionless particle parameter	(-)
e_b	efficiency factor of bed load (= 0.1-0.2)	(-)
e_s	efficiency factor of suspended load (= 0.01-0.02)	(-)
F	shape factor in van Rijn formula	(-)
g	acceleration of gravity	(m/s^2)
h	water depth	(m)
k_s	effective bed roughness height of Nikuradse	(m)
κ	Von Karman constant (= 0.4)	(-)
K	weighing factor for the i-th sample	(-)
L_D	damping length	(m)
L_P	wave length	(m)
m, n	coefficients in Ackers and White formula	(-)
N	sum of the weighing factors at a vertical	(-)
s	Sediment transport per unit width	(m^2/s)
s_b	bed load transport	(m^2/s)
s_s	suspended load transport	(m^2/s)
s_t	total load transport	(m^2/s)
Q	measured water discharge	(m^3/s)
Q_c	assessed discharge	(m^3/s)
s (= ρ_s/ρ)	specific density	(-)
I	slope of energy gradient	(-)
i	slope of energy gradient	(-)
T	dimensionless bed-shear stress parameter in van Rijn formula	(-)
u	depth averaged velocity	(m/s)
u_*	current-related bed-shear velocity	(m/s)
u_i	velocity at the i th measuring point	(m/s)

u_{cr}	depth-averaged velocity at initiation of motion	(m/s)
w	settling velocity of the sediment load	(m/s)
w_r	settling velocity for a representative sediment size	(m/s)
w_s	fall velocity (based on d_{50} of bed material)	(m/s)
Y	particle mobility parameter	(-)
Y_{cr}	critical particle mobility parameter	(-)
z	suspension number	(-)
z_o	zero velocity level	(m)
z_i	height above the bed of point i	(m)
α_2	coefficient according to Parker and Klingeman	(-)
β	ratio of sediment and fluid mixing coefficients	(-)
$\tan\beta$	bottom slope in Bagnold equation	(-)
γ_s	specific weight of the sediment particle	(kg/m^3)
γ_f	specific weight of the fluid particle	(kg/m^3)
ρ	fluid density	(kg/m^3)
ρ_s	sediment density	(kg/m^3)
Δ	relative density	(-)
$\tan\phi$	dynamic friction coefficient(= 0.6)	(-)
ϕ_{50}	dimensionless flow parameter in Parker and Klingeman formula	(-)
θ	dimensionless Shields parameter	(-)
θ'	dimensionless shear stress parameter based on grain roughness	(-)
μ	bed-form factor or efficiency factor	(-)
μ	dynamic viscosity of the fluid	(kg/sm)
τ_b	over-all current-related bed-shear stress	(N/m^2)
τ_b'	effective bed-shear stress	(N/m^2)
$\tau_{b.cr}$	critical bed-shear stress according to Shields	(N/m^2)
ν	kinematic viscosity coefficients	(m^2/s)
λ_s	adaptation length of the sediment transport and bed topography	(m)
λ_w	adaptation length of the main flow	(m)
Ψ	dimensionless sediment transport	(-)
Ψ	stratification correction in van Rijn formula	(-)
σ_s	geometric standard deviation of bed material	(-)
σ	standard deviation	(-)

Acronyms and abbreviations

ASCE	: American Society of Civil Engineers
BUET	: Bangladesh University of Engineering and Technology
BWDB	: Bangladesh Water Development Board
FAP	: Flood Action Plan
GJP	: Ganges-Jamuna-Padma (mathematical model)
RSP	: The River Survey Project (= FAP 24)
WMO	: World Meteorological Organisation

1 Introduction

The River Survey Project (FAP24) was initiated on June 9, 1992. The project is executed by the Flood Plan Coordination Organization (FPCO) presently Water Resources Planning Organization (WARPO) under the Ministry of Irrigation, Water Development and Flood Protection, with funding by the Commission of the European Communities. Consultants are DELFT-DHI joint venture in association with OSIRIS, Hydroland and approtech. Project supervision is undertaken by a Project Management Unit with participation by FPCO, a Project Adviser and a Resident Project Adviser.

The objective of the project is to establish the availability and accurate field data as a part of the basis for the FAP projects, as well as providing input for other planning, impact evaluation and design activities within the national water resources and river engineering activities.

The project consists of three categories of activities:

- a survey component, comprising a comprehensive field survey programme of river hydrology, sediment transport and river morphology
- a study component, comprising investigations of processes and effects within river hydrology, sediment transport and river morphology
- a training component.

The rivers in Bangladesh are alluvial in nature. a proper quantification of the extent of erosion, deposition, and changes in channel form of such alluvial courses has been a subject of considerable research, and still is. In the water development sector, for planning and design, it is required to predict future changes of the river morphology. Hereby, the sediment transport is a main feature. Therefore, a systematic knowledge about river flow and sediment transport is of utmost importance from the standpoint of improved planning and design of water development projects, and knowledge is essential about the factors that determine the sediment transport in a given flow.

This report deals with the development of a sediment transport equation applicable for the major alluvial rivers in Bangladesh. The developed equation is compared with well known sediment transport prediction formulae against the same data. These analyses were carried out for the Jamuna, the Ganges and the Padma Rivers. Hereby, certain inconsistencies were noticed in the basic material of sediment transport data. These inconsistencies were further examined by 1-dimensional modelling.

Additionally, a theoretical investigation have also been carried out in order to examine the magnitude of the exponent 'n' of the flow velocity in sediment transport prediction formulae applicable for alluvial rivers.

Data were taken from the field sheets on sediment and discharge measurements carried out by Surface Water Hydrology II, Bangladesh Water Development Board (BWDB). The quality of these data has been assessed with respect to the morphological and hydraulic conditions of the gauging stations (Chapter 5). The performance of a few selected sediment predictors has been investigated relative to measured sediment transport data at Bahadurabad, Hardinge Bridge, and Baruria gauging stations on the Jamuna, Ganges and Padma Rivers, respectively (Section 5.3). The selection of the predictors is based on experience from other research and study projects in Bangladesh and abroad. Additionally, the power coefficient of the flow velocity of the sediment transport predictors has been examined.

This report describes an activity within the study programme of the River Survey Project (DELFT/DHI, 1995), namely Study Subject 2: Sediment transport, topic 2.2: Sediment transport predictors. This study has been carried out and reported by Saleem Mahmood, River Morphologist-FAP 24 and Mr. Krishna Chandra Dey, River Engineer-FAP 24.

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2 Background, objective and approach

2.1 Background

'What sediment transport formula should we recommend to estimate the total sediment discharge in river reaches?' This question always arises from hydraulic and environmental engineers who are engaged in various sediment-control projects. The question has been examined within the River Survey Project, which is dealing with a wide range of objectives related to sediment-discharge prediction in the natural movable-bed rivers in Bangladesh.

Numerous theoretical and semi-empirical approaches have been developed for the calculation of sediment transport rates (for example Einstein 1950, Yalin 1963, Bagnold 1966; Ackers & White 1973, van Rijn 1984; Engelund-Hansen 1967; Yang 1973, Parker and Klingeman 1983; Pantal 1971). The work has mainly been based on observations in laboratory experimental flumes and in small streams. However, a few attempts have been made to test the applicability of the relationships in very large sand bed rivers, where the river morphology may strongly control the transport rates. FAP 1 and the Jamuna Multi-purpose Bridge Project have used the Engelund-Hansen and van Rijn predictors for estimation of sediment transport in Jamuna River. In a paper, Lukanda et al. (1992) present results obtained with four formulae using annual flow discharge data from Zaïre River, the second largest river in the World.

For general application in morphological studies, either by numerical analysis or by scale modelling, it is attractive to use a schematised sediment transport formula in a form which can be defined as the sediment transport per unit width as a function of the flow velocity to the power 'n', or the dimensionless sediment transport as a function of the Shields parameter to the power n/2. Most prediction formulae can be converted to such a schematised form. In general, the power 'n', which is the exponent of the flow velocity, varies between 3 and 7 and even higher (higher values for a lower transport rate), with 'n' equal to 4 to 5 for high sediment transport rates (Breusers, 1988). The exponent 'n' depends on the Shields parameter, which is a dimensionless shear stress parameter.

In most of the prediction formulae, the exponent 'n' plays an important role in estimating the sediment transport. The importance of 'n' is demonstrated by Jansen et al. (1979), Struiksma et al. (1985), Struiksma and Klaassen (1988), and Struiksma and Crosato (1989). Hereby, it has been shown that point bar formation in channel bends, bar dimensions, transition between meandering and braiding, and the braiding index are closely linked to the exponent 'n'. Besides, in morphological computations, the exponent 'n' influences the prediction of the river bed response by any morphological model. de Vries (1986) demonstrated that the slope and the flow depth of an alluvial river are dependent on the value of 'n'. While the importance of 'n' is well understood by the predictors, its significance in each prediction formula is not well explained, and in most of the prediction formulae, the range of 'n' and its sensitivity to different hydraulic parameters are not known.

Erosion and deposition in alluvial rivers are characterized by the spatial variation in sediment transport, in accordance with the non-linearity of the sediment transport formula which describes the relation between the flow characteristics and the quantities of the bed material being moved. The importance of this schematised transport equation is that such simplified relationship can be combined with other equations to arrive at analytical expressions that provide increased insight into the morphological behaviour of an alluvial river.

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Unless sediment measurements in the field are done properly, it would be extremely difficult for an engineer to choose an appropriate sediment transport prediction formula. In this study, it is found that the measurements at the constriction of the Hardinge Bridge on the Ganges river show a scatter in the data and values, and predict different sediment transport upstream of the bridge. Similarly, the morphology at the confluence of the Jamuna and the Ganges may influence the sediment transport at Baruria on the Padma, a few km down of the confluence.

2.2 Objective

The main objectives of this study are two-fold: Firstly, the development of an appropriate schematized sediment transport formula for the alluvial major rivers in Bangladesh, and, secondly, exploration of the exponent 'n' of the generalized sediment transport formula '$s=mu^n$'.

Additionally, an assessment has been made of the quality of the BWDB data used in the analysis.

These objectives can be elaborated in more detail as follows:

- To study the sediment transport phenomena in the major rivers in Bangladesh and to verify the applicability of a 'normal' transport equation. This equation does not take sediment plumes into account, i.e. over-loading and under-loading of sediment into the system. The normal schematized sediment transport formula reads:

$$s = m\,u^n \qquad (2.1)$$

- To determine and explain the exponent n in this equation 1, a parameter determining the morphological behaviour

- To determine of the values of n for selected prediction formulae widely used and suitable for the alluvial rivers in Bangladesh.

- To derive a sediment transport formula for the alluvial rivers in Bangladesh, which can be used in various studies

- To identify any additional processes to be taken into account in addition to the normal shear stress

2.3 Approach

The study has compared measured values of bed material transport in the Jamuna, Ganges and Padma rivers in Bangladesh to sediment transport prediction formulae (such as Engelund-Hansen, van Rijn, Ackers and White, Bagnold, and Yang) and has considered the possibility to adjust one of these formulae to get a sediment transport predictor with a good validity. Moreover, a new prediction formula has been derived from the measured data without the use of any existing formulae.

Hereby, it has been an aim to simplify the existing sediment transport prediction formulae numerically and analytically, in order to derive 'n', the power of non-linearity, and to verify the sensitivity on 'n' with the change of grain size, water level slope etc. in the sediment transport prediction formulae.

Historical BWDB data were used. Throughout the study, the grain size is limited to the coarse sediment i.e. greater than 63 micron.

A detailed analysis has been made for the gauging station at Bahadurabad, and a less detailed analysis for the gauging stations at Hardinge Bridge and Baruria.

A one-dimensional mathematical model was used to verify inconsistencies in the BWDB sediment transport data.

3 Applicability of prediction formulae

3.1 General

In the literature, the performance of existing sediment transport prediction formula is not very well described relative to measured flow and sediment data in natural rivers. Most of the formula are tested under laboratory conditions.

Several prediction formulae exist, but it is not possible to determine positively which one gives the more realistic result. A formula that predicts sediment discharges adequately for one river, does very poorly for an other one. For example, the Inglis-Lacey formula (Inglis 1968) seems to predict the sediment transport for the Colorado River quite well, while estimates for Niobrara river are extremely high, even though these two rivers have very similar energy slopes and a similar geometric mean diameter of the grain size.

To ease this difficulty, many of the commonly used transport formulae have been tested by different researchers over a wide range of field and laboratory data. Key studies have been published by:

(i) The ASCE Task Committee for preparation of sedimentation manual (1971)
(ii) White et al. of the Hydraulic Research Station (HRS), Wallingford (1973)

Below, a review is given of selected publications, and different sediment transport equations are discussed.

3.2 Literature overview

The Task Committee (1971) have studied thirteen formulae and observed that the Colby (1964), Engelund-Hansen (1967), and Tofaletti (1969) formulae gave consistently better agreement than others. However, in presenting their results, they continued that 'these are not conclusions but recommendations or suggestions presented for guidance until more complete studies can be made'. On the other hand, Wallingford's studies are the most comprehensive and recent. They have tested nineteen transport formulae against about 1000 laboratory and 270 field measurements. They divided the theories into four groups, namely A, B, C and D according to general performance. It was found that out of the nineteen theories, the total load formulae such as Engelund-Hansen (1967) and Ackers & White (1973), and the Rottner (1959) bed load formula, are the most reliable and are applicable for a wide range of flow conditions and sediment characteristics.

Based on a stream power concept, Yang (1977) developed a sediment transport equation. He compared his equation with ten other sediment transport equations against 154 sets of field and 1093 sets of laboratory data. He found that Yang's equation is superior to others. According to his evaluation, his equation can replace Shen-Hung, Ackers and White, Engelund-Hansen and the other equations.

Eleven sediment predictors were tested against field data collected from the Sacramento River in California, where the bed grain size varies from fine to coarse sand (Nakato T., 1990). They include the Ackers-White, Einstein-Brown, Engelund-Fredsoe, Engelund-Hansen, Inglis-Lacey, Karim, Meyer-Peter and Mueller, van Rijn, Schoklitsch, Tofaletti, and Yang formulae. It was found that the computed values deviate significantly from the measured values except for a very few cases. A comment was made which is that 'the test results clearly demonstrate how difficult a task it is to predict sediment transport in natural rivers'.

The predictive capability of the sediment transport formulae of Ackers-White, Engelund-Hansen and van Rijn at high flow velocities was investigated by Voogt et al. (1991). The total load transport at high velocities over a fine sand bed (100-400 μm) was studied by analysing data collected from various literature sources and new flume and field measurements. Flume experiments were performed to measure the flow velocity and sand concentration profiles in a self-eroding flow initially free of sediment. The flow velocities were measured up to 2.7 m/s. A two-dimensional vertical mathematical model for suspended sediment was applied to compute the equilibrium transport rates. Finally, the predicted transport rates were compared with measured transport rates under flume and field conditions. It was found that the formulae of Engelund-Hansen and van Rijn predict transport rates with reasonable agreement against the measurements.

Yang and Wan (1991) made a comparison of the over-all accuracy of the prediction formula. They considered different ranges of sediment concentration, Froude number and slope for seven bed-material load formulae in their analysis. Four formulae that can compute bed-material load transport by size fractions were used to determine the particle size distribution of the bed-material in transportation. A total of 1119 sets of laboratory data and 319 sets of river data in the grain size range were used to evaluate and compare the accuracy of these formulae. The over-all accuracy of the formulae, when applied to laboratory flumes, was as follows, in descending order: Yang, Engelund-Hansen, Ackers and White (d_{50}), Laursen, Ackers and White (d_{35}), and the Colby, Einstein and Toffaleti formulae. The accuracy, when applied to natural rivers, was, in descending order: Yang, Toffaleti, Einstein, Ackers and White (d_{50}), Colby, Laursen, Engelund-Hansen and Ackers and White (d_{35}). However, these ratings may vary depending on the values of sediment concentration, Froude number and slope of the data used in comparison. The study also indicated that Yang's formula by size fractions can accurately predict the size distribution of bed-material in transportation, while Einstein's hiding and lifting correction factors overcorrected the effect of non uniform size distribution of bed material on the total bed-material transport.

Lukanda et al. (1992) studied the applicability of sediment transport theories for the Zaïre River. The objective of their study was to identify a suitable sediment transport formula for the analysis of the sediment problems experienced in the inner delta of the Zaïre River's maritime reach. They used four formulae, which were Schoklitsch, Shields, Meyer-Peter & Muller and Bagnold. Bagnold's approach was adapted for application for predicting morphologic changes at local stations.

Van den Berg et al. (1993) studied the prediction of suspended bed material transport in flows over silt and very fine sand for the Huanghe (Yellow) river. Three equilibrium sand transport formula were tested, namely Ackers & White (1973), Engelund-Hansen (1967) and van Rijn (1984b). According to this study, the best results were obtained with the van Rijn formula, except at low flow stages, where better results were produced by the Engelund-Hansen equation. The Ackers & White formula seriously over-predicted the measured values. Some modifications of the van Rijn formula were proposed for application in flows over very fine sand and silt. In a verification analysis, it was demonstrated that these modifications slightly improve its predictive strength.

3.3 Sediment transport and morphological studies in Bangladesh

Several studies have been done in the past on the application of sediment prediction formula in Bangladesh. A few of these are mentioned below:

In FAP 1 (1993, Annexure 1), two prediction formulae, the Engelund-Fredsoe and the van Rijn formulae, have been used to calculate the sediment transport in the Brahmaputra River, particularly for mathematical modelling of bend scour. The two models predict slightly larger transport rates than the Engelund Hansen model, which is a total load model.

The measured transport for Jamuna River has been compared with the Engelund-Hansen formula for at-a-station conditions (G. J. Klaassen and K. Vermeer, 1988). It was found that the measured sediment transport of the period 1968-1970 can be well represented by the Engelund-Hansen formula multiplied by 2, when channel dimensions are determined by analysis of the measured BWDB cross-sections.

Bari and Alam (1979) worked with sediment prediction using data from the Ganges River at Hardinge Bridge. This study indicates that a sediment transport rate by Colby equation shows a closer match with the measured values, and that the Engelund-Hansen formula gives consistent results.

Waliuzzaman (1986) selected five equations, namely Yang (1973), Ackers and White (1973), Engelund-Hansen (1967), Hossain (1985) and Mantz (1983), for analysing the sediment transport in the river Gorai-Madhumati. He concluded that the Hossain and the Engelund-Hansen equations produced better predictions of sediment transport for that river.

In connection with the design of the Teesta Barrage Project, BUET-BWDB (1988) conducted a study on the sediment transport characteristics of the river Teesta. They selected Yang, Engelund-Hansen and Colby's equations and concluded that Yang's equation was best for the purpose.

More recently, some well-known sediment transport prediction formulae have been utilized and compared for the application for morphological studies in Bangladesh. Galappatti (1993) used the Engelund-Hansen formula for the development of 1-D morphological model for the Jamuna River. Islam (1994) used the Ackers and White, Engelund-Hansen, Engelund-Fredsoe and van Rijn formulae for studying the bed level changes for the Ganges river by using a one-dimensional morphological model (MIKE 11). It has been found that the Engelund-Hansen formula is reasonably good for predicting the bed level changes.

Based on measured data, Matin and Mohiuddin (1994) attempted to develop a schematised transport equation applicable for the north-west regional rivers of Bangladesh. The relationship obtained was compared with the schematised form of the Engelund-Fredsoe (1976) and van Rijn (1984) equations. In this study, the van Rijn predictor was found comparatively suitable for the prediction of sediment transport.

3.4 Literature on the exponent 'n'

Very few analyses have been made on the importance of the exponent 'n' of the velocity in the power law sediment equation. The understanding of this issue is at a preliminary stage. The range of n has been computed for very few prediction formula. In the following, some available literature is summarized.

3.5 Explanation of different sediment transport equations

Sediment transport formulae are describing the relation between the flow characteristics and the quantities of bed material being moved. Quite a number of sediment transport formula are in use, as exemplified below:

- In a RSP Study on 'Sediment Rating Curves and Balances', sediment rating curves are produced with equation 3.1:

$$S=AQ^{B} \tag{3.1}$$

The curve produced by this equation is generally referred to as a sediment rating curve, which is an exponential function which can be determined either by regression analysis or from a graph with the data points (discharge vs. sediment transport) on a logarithmic scale. This curve is widely used to Oestimate the sediment concentration or the sediment transport for periods where discharge data are available but sediment transport data are not. A sediment rating curve between S and Q assumes an unique relationship between the average flow velocity in a cross-section and the shear stress at the river bed. This unique relationship exists in a steady uniform flow with a fully developed boundary layer. In such flows, the vertical flow velocity profile is described with a logarithmic profile or a power law profile. This unique relationship requires more or less prismatic cross-sections with only one channel in a cross-section of the river.

However, in an accelerating or decelerating flow, deviations in a sediment transport rating curve can be expected. These types of flow occur in bends, near bifurcations and confluences of the channel of a braiding river, and during the rising and receding limbs of the hydrograph. Consequently, the sediment rating curves for the main river system in Bangladesh appear as regression lines fitted to strongly elongated clouds of data points.

The real picture of sediment transport can be seen with a relation between the sediment concentration 'c' as the dependent variable and the liquid discharge 'Q' as an independent variable. The relation between the sediment discharge 'S' and the liquid discharge 'Q' is a spurious correlation, because the sediment discharge 'S' is the multiplication product of the sediment concentration 'c' and the liquid discharge 'Q' in a width increment between two measuring verticals. This relation gives a better correlation than the previous one. Transferring this relation into logarithmic values gives at the one hand a much better correlation, but on the other hand, it is more spurious.

- Another simplified formula used in the computation of sediment transport is the direct power regression equation between the sediment transport and velocity, which reads as:

$$s = m\ u^{n} \tag{3.2}$$

This formula considers the velocity as the only hydraulic parameter. This relation is also spurious, as the sediment discharge per meter width 's' is a multiplication product of the depth average velocity 'u' and the depth average sediment concentration 'c' in a measuring vertical.

- Spatial variations in the sediment transport determine deposition and erosion due to deceleration and acceleration respectively, and this is fully governed by the non-linearity of the sediment transport formula. This can be represented by a schematized formula which reads as equation (2.1).

In general, the power n varies between 3-6 for the existing flow conditions of the alluvial rivers in Bangladesh. The above equation describes the relation between the flow characteristics and the quantities of bed material being moved. The advantage of using this equation is that such a simplified relationship can be combined with other equations to arrive at analytical expressions that provide increased insight into the morphological behaviour of an alluvial river. The equation can be elaborated for the over-all condition of transport as follows:

$$\frac{s}{\sqrt{g\Delta d_{50}^{3}}} \propto f[(\theta - \theta_{cr})^{\frac{n}{2}}] \qquad (3.3)$$

In Bangladesh, usually $\Theta >> \Theta_{cr}$ and the equation can be simplified to:

$$\frac{s}{\sqrt{g\Delta d_{50}^{3}}} \propto f(\theta^{\frac{n}{2}}) \qquad (3.4)$$

The above equation can be re-written as:

$$\psi \propto f(\theta^{\frac{n}{2}}) \qquad (3.5)$$

where

$$\theta = \frac{hi}{\Delta d_{50}} = \frac{u^2}{C^2 \Delta d_{50}} \qquad (3.6)$$

$$\psi = \frac{s}{\sqrt{g\Delta d_{50}}} \qquad (3.7)$$

The exponent n depends on the Shields parameter, Θ, which is a dimensionless shear stress parameter.

In this study, equation (3.5) is used for deriving a sediment transport prediction formula, as this equation has similarities with most of the available sediment transport prediction formulae and it incorporates the influential hydraulic and morphological parameters. Moreover, this equation is not spurious, as both the dependent and the independent terms are divided by constant parameters.

3.6 Selection of prediction formula

Most sediment transport formulae can be schematized to a simple relation between 's' (sediment transport per unit width) and 'u' (depth average velocity), as in equation (2.1). In this study, several prediction formulae have been analytically verified for explanation of the significance of the power of the flow velocity and for derivation of a suitable prediction formula for the rivers in Bangladesh. The selection of prediction formulae has been based on the following criteria:

- suitability for the conditions in the rivers in Bangladesh
- ability to explain hydrological and morphological phenomena
- possibility to derive the exponent n of the flow velocity
- estimation of the sediment transport as a function of the bed shear stress, a parameter influenced by the accelerating and decelerating flow condition
- being widely used in the major rivers in Bangladesh and abroad

No data on the bed load transport was available for analysis. Therefore, prediction formulae for suspended sediment transport only were preferred for comparison with the measured data. However, in some cases, prediction formulae for total load transport were considered as well.

For exploration of the exponent 'n', such prediction formulae were considered that were regarded as important and as related to the rivers of Bangladesh.

The prediction formulae identified in this way are listed in Table 3.1

Prediction formula	Type of formula	Sediment transport	exponent 'n'
Ackers and White (1973)	Total load	yes	yes
Bagnold (1966)	Suspended load	yes	no
Bagnold (1966)	Total load	no	yes
Engelund-Hansen (1967)	Total load	yes	yes
Yang (1973)	Total load	yes	yes
Colby (1957)	Total load	no	yes
Van Rijn (1984)	Suspended load	yes	no
Van Rijn (1984)	Total load	no	yes
Van Rijn (1984)	Bed load	no	yes
Van Rijn (1984) Simplified	Bed load	no	yes
Meyer-Peter and Mueller (1948)	Bed load	no	yes
Parker and Klingeman (1983)	Bed load	no	yes

Table 3.1: Different prediction formulae used in the analysis

The occasional consideration of total load prediction formulae is justified, because, as we know, the influence of bed load transport is negligible in the over-all sediment transport in the rivers in Bangladesh (FAP 1, 1992, FAP 21/22).

3.7 Data organization

This study concentrates on data from the Jamuna, the Ganges and the Padma Rivers. Data from three sources have been collected: Field data from the River Survey Project (FAP 24) and from the Bangladesh Water Development Board (BWDB), and laboratory data from various literature.

The hydraulic data and sediment data used in the analysis are described below. Locations are shown in Figure 3.1.

Data for sediment transport predictor analysis

Extensive flow and suspended bed material transport data were measured by BWDB at the Bahadurabad, Hardinge Bridge and Baruria stations. The available field sheets on sediment measurements and discharges were collected for the periods 1983-1987 for Bahadurabad, 1990-1994 for Hardinge Bridge, and 1986-1994 for Baruria. The data used for the derivation of a sediment transport formula are shown in Table 3.2. This table lists bed material samples collected for size analysis by the River Survey Project, and the calculated over-all water surface slopes for the major rivers.

Name of river	Gauging station	Grain size of bed material River Survey Project FAP 24							Over-all water level slope 1993-1994
		No. of samples	Collection period	D_{16} mm	D_{35} mm	D_{50} mm	D_{84} mm	D_{90} mm	
Jamuna	Bahadurabad	56	1993-1994	0.130	0.160	0.220		0.340	0.000075
Ganges	Hardinge Bridge	50	1993-1994	0.100	0.120	0.150	0.180	0.210	0.000055
Padma	Baruria	30	1993-1994	0.100	0.124	0.140	0.185	0.220	0.00004

Table 3.2: Grain size and over-all slope used in the analysis of sediment transport prediction formula

Data used for derivation of the exponent 'n'

The ranges of test data used in the derivation of the existing prediction formulae are given in Table 3.3. Most of these are based on flume test results. In addition to these data, a depth range as expected in Jamuna River and a Froude number as expected in an alluvial river have been considered in order to make graphical relations for the exponent 'n'. These data are presented in Table 3.4.

Sediment prediction formula	depth h (m)		flow velocity u (m/s)		energy slope i (-)		grain diameter d_{50}(mm)	
	min	max	min	max	min	max	min	max
Ackers and White	-	-	-	-	-	-	0.04	0.40
Bagnold	-	-	-	-	-	-	0.79	30.00
Engelund-Hansen	0.09	0.33	0.25	1.60	-	-	0.19	0.93
Parker and Klingeman	nk	nk	nk	nk	nk	nk	nk	nk
Van Rijn	1.00	20.00	0.50	2.50	-	-	0.10	2.00
Yang	0.01	15.20	0.23	1.97	0.0043	0.0279	0.14	1.71
Meyer-Peter and Mueller	0.01	1.20	-	-	0.0004	0.0200	0.40	30.00

nk = not known

Table 3.3: Range of test data used for deriving the prediction formulae

The median particle diameter of the bed material for the main rivers in Bangladesh is in the order of 0.14 mm to 0.30 mm. For the derivation of Bagnold and Meyer-Peter and Mueller transport formulae, a larger range of grain sizes was used (as shown in Table 3.3) than the median grain diameter for the major rivers in Bangladesh. On the other hand, the Meyer-Peter and Mueller and Yang sediment transport formula were developed on the basis of much steeper slopes than the slopes of the alluvial rivers in Bangladesh, which, in general, varies between 4 and 9 cm/km.

Type of sediment transport formula	Depth range, h (m)	Froude number range, Fr (-)	Sensitivity analysis	
			Slope range, i (cm/km)	Grain diameter range, d_{50} (mm)
Total load	0.50-20.00	0.20-0.40	5-10	0.12-0.35
Bed load	2.00-20.00	0.088	-	-

Table 3.4: Data used for the sensitivity analysis of the exponent 'n'

4 The exponent 'n' of the flow velocity

4.1 Introduction

In most prediction formulae, the exponent 'n', the power to the flow velocity, plays an important role in estimating the sediment transport. Although the importance of this exponent is well understood by the researchers, its significance in each prediction formula is not well explained, and its range is unknown in most prediction formulae.

In this chapter, an analytical solution was applied for derivation of 'n'. A numerical solution was used for complex prediction formulae, where an analytical solution is not practical.

A sensitivity analysis was also carried out relative to different hydraulic parameters.

4.2 The importance of 'n' in Schematised Transport Equations

The importance of the exponent 'n' has been demonstrated by Jansen (1979), Struiksma et al. (1985), Struiksma and Klaassen (1988), and Struiksma and Crosato (1989). They have shown that point bar formation in channel bends, bar dimensions, transition between meandering and braiding, and the braiding index are closely linked to the exponent 'n'. Besides, in morphological computations, the exponent 'n' influences the prediction of the river bed response by a morphological model. In rivers with alluvial sediment, the slope of the channel bed and the flow depth depend on the value of 'n', as demonstrated by Vries (1986).

An approximate relationship between 'n' and the Shields parameter (θ) has been derived for the Meyer-Peter and Mueller transport prediction formula, as corrected for form roughness (Struiksma and Klaassen, 1988). The relationship is presented in Figure 4.1, which shows that the Shields parameter (θ') decreases with an increased value of 'n' and vice versa. This evaluation indicates that 'n' is not always constant, but is a function of (θ'). The corrected expression for θ' reads as:

$$\theta' = \frac{C^2}{C_{90}^2} \cdot \frac{u^2}{C^2 \Delta d_{50}} = \mu\theta \qquad (4.1)$$

where

$$C_{90} = 18 \log \left(\frac{12h}{d_{90}}\right) \qquad (4.2)$$

$$C = 18 \log \left(\frac{12h}{k_s}\right) \qquad (4,3)$$

Struiksma and Klaassen (1988) studied the equilibrium bed topography in curved channels and developed the criteria for the spatial amplification of steady bed level perturbations. The results were used to derive a prediction for the threshold between meandering and braiding channels as shown in Figure 4.2. Here, the exponent 'n' was found to be an important factor for deriving the threshold values. They showed that the wave length L_p and the damping length L_D of perturbations depend on the ratio λ_s/λ_w, where λ_s is the adaptation length of the sediment transport and the bed topography development, and λ_w is the adaptation length of the main flow. This ratio was again related to the exponent 'n', as shown in Figure 4.3. The criteria derived for different planforms of the rivers are as follows:

$\lambda_s/\lambda_w < \lambda_s/\lambda_w^*$	:	meandering
$\lambda_s/\lambda_w^* < \lambda_s/\lambda_w < \lambda_s/\lambda_w^{**}$	:	transition between meandering and braided
$\lambda_s/\lambda_w^{**} < \lambda_s/\lambda_w$	:	braided

Here, λ_s/λ_w^* and λ_s/λ_w^{**} are the lower and upper limits of the harmonic range. The values of λ_s/λ_w^* and λ_s/λ_w^{**} depend on the value of the exponent 'b', according to the following relationships:

$$\frac{\lambda_s^*}{\lambda_w} = \frac{2}{n-3} \tag{4.4}$$

and

$$\frac{\lambda_s^{**}}{\lambda_w} = [\frac{1}{2}(n+1)-\frac{1}{2}((n+1)^2 - 2(n-3))^{0.5}]^{-1} \tag{4.5}$$

Here, it is important to mention that Figure 4.1 was used for the transformation of θ' to the exponent 'n'. The proposed criterion for planform classification merely consists of computing the values of λ_s/λ_w and 'n', and comparing these with the appropriate values of λ_s/λ_w^* and λ_s/λ_w^{**}. Figure 4.3 can also be used for this purpose. In this figure, the indices B and M indicate whether the channel pattern is braided or meandering. Thus it is necessary to determine the accurate value of the exponent 'n' for a fair prediction of the channel pattern.

Struiksma et al (1985) have also shown the importance of the exponent 'n' for bed deformation, which is because 'n' affects the rate of damping in such a way that a decrease of 'n' leads to a considerable increase of damping.

For a constricted or a widened river section, the influence of 'n' on the main parameters of flow and sediment transport can be easily demonstrated. Consider a river with fixed banks and a constant discharge Q. The question is raised to what extent the depth will increase if the width is decreased. Indicating the original values of the parameters with subscript 'o' and the new ones with '1', the following basic equations describe in principle the flow and the sediment transport along a flow line (using depth instead of hydraulic radius):

For discharge:	Continuity equation:	$Q_o = Q_1$
	Motion equation:	$Q = B.C.h^{3/2}.I^{1/2}$
For sediment transport:	Continuity equation:	$Q_{S1} = Q_{So}$
	Motion equation:	$Q_S = B.m.u^n$

Hereby, in fact, two equilibrium situations are compared. These are the existing and the final future situation. For a restricted reduction in the width, the assumption is $C_1 \approx C_o$, as well as $d_1 \approx d_o$. This leads to the conclusion that 'm' and 'n' are also the same in the two situations. It can be shown that:

$$\frac{h_1}{h_o} = \left(\frac{B_o}{B_1}\right)^{\frac{n-1}{n}} \tag{4.6}$$

and

$$\frac{I_1}{I_o} = \left(\frac{B_1}{B_o}\right)^{1-\frac{3}{n}} \tag{4.7}$$

A graphical representation of equations (4.6) and (4.7) is shown in Figure 4.4, which indicates the effect of a constriction of the river width. As an example, for 'n' = 3, the slope does not change, but the depth will change. However, for higher values of 'n', both the slope and the depth become influenced.

4.3 Assessment of 'n'

The value of 'n' can be assessed straightforward analytically from any sediment transport equation, where the exponent of the flow velocity appears directly. For example, the Engelund-Hansen prediction formula directly yields the value of 'n' equal to 5. The same is the case for the simplified prediction formula of Colby, where 'n' equals 3.1.

However, there are other sediment transport prediction formulae where the exponent of the flow velocity does not appear directly. In order to assess the exponent 'n' of such prediction formulae, two possible ways have been identified in this study.

The exponent 'n' of the flow velocity can be assessed by using the following relation:

$$n = \frac{ds}{du} \cdot \frac{u}{s} \tag{4.8}$$

Equation 4.8 follows from the schematised sediment transport equation, which can be expressed as:

$$s = m\, u^n \tag{4.9}$$

Here, ds/du is the derivative of the sediment transport per unit width with respect to 'u', 'u' is the flow velocity, and 's' is the sediment transport per unit width. This method can be applied to predictors like the ones of Ackers and White (1973), Bagnold (1962), Meyer-Peter and Mueller (1948), Parker and Klingeman (1983), van Rijn (1984), and Yang (1973).

However, this analytical expression can only be used when it is possible to derive the differential ds/du from the sediment transport prediction formula. There are some complicated transport equations where this is not straightforward. In such cases, $\Delta s/\Delta u$ can be determined numerically, and 'n' can be found by the following expression:

$$n \approx \frac{\Delta s}{\Delta u} \cdot \frac{u}{s} \qquad (4.10)$$

4.4 Derivation of 'n' from prediction formulae

Differentiation of equation (4.9) with respect to 'u' gives

$$\frac{ds}{du} = \frac{n}{u} (m\ u^n) = \frac{n}{u} s \qquad (4.11)$$

Hereby, 'n' can be derived from the selected prediction formula, whereafter 'n' can be related to the Shields parameter (θ). This parameter in turn reflects most of the hydraulic parameters (e.g slope, velocity, grain size etc.) that influence the sediment transport.

The detailed derivation of 'n' from each prediction formula is described in Annexure 1.

4.5 Derivation from selected Total Load Formula

Six total load prediction formulae have been selected for derivation of 'n'. These are: Ackers and White, Bagnold, Colby, Engelund-Hansen, van Rijn, and Yang. The criteria for selecting these equations are stated in Section 5.1. Below, the details of the derivation for each prediction formula are outlined.

a. Ackers and White (1973): Total load formula

Assumption: log ($10h/d_{35}$) considered as constant

$$n = 1 + \frac{m'}{(1 - \frac{Y_{cr}}{Y})} \qquad (4.12)$$

where

$$Y = (\frac{\sqrt{g}}{C})^{(n'-1)} \sqrt{\theta d_{50}/d_{35}} \ (\frac{1}{5.66 \log(10h/d_{35})})^{(1-n')} \qquad (4.13)$$

b. Bagnold (1966): Total load formula

Assumption: $\tan\beta << \tan\phi$ and $\tan\beta << w_s/u$

$$n = 3 + \frac{1}{1 + \dfrac{e_b}{e_s(1-e_b)\tan\phi}\,\dfrac{w_s}{C\sqrt{\Delta d_{50}\theta}}} \tag{4.14}$$

c. Engelund-Hansen (1967): Total load formula

The power of the flow velocity appears directly:

$$n = 5 \tag{4.15}$$

d. Colby (1957): Total load formula

The power of the flow velocity appears directly in the simplified formula:

$$n = 3.1 \tag{4.16}$$

e. Van Rijn (1984): Total load formula (original form)

Assumption: B is constant, where

$$B = \frac{1}{C^2 I}\left[0.015\, F\, \frac{d_{50}}{a}\, \frac{1}{D_*^{0.3}} + 0.10\, \frac{d_{50}}{D_*^{0.3}}\, \frac{1}{Ch}\sqrt{\frac{g}{\theta}}\right] \tag{4.17}$$

$$n = 3 + \frac{3}{1 - \dfrac{\theta_{cr}}{\theta}\left(\dfrac{C'}{C}\right)^2} \tag{4.18}$$

f. Van Rijn (1984) : total load formula (simplified form)

Assumption: B is constant, where

$$B = \left(C\sqrt{\frac{\theta}{g}}\right)^{2.4} \frac{1}{C^2 i}\left[0.012\, \frac{d_{50}}{h}\left(\frac{1}{D_*}\right)^{0.6} + 0.005\left(\frac{d_{50}}{h}\right)^{1.2}\right] \tag{4.19}$$

$$n = 3 + \frac{2.4}{\dfrac{C\sqrt{\Delta d_{50}\theta}}{u_{cr}} - 1} \tag{4.20}$$

g. Yang (1973): Total load formula

Assumption : $u_{cr} << u$

$$n = 6.397 - 1.227\log\left(\frac{w_s d_{50}}{\nu}\right) - 0.942\log\left(\frac{\sqrt{(g\Delta d_{50}\theta)}}{w_s}\right) \qquad (4.21)$$

4.6 Derivation from selected bed load formula

a. Van Rijn (1984): Bed load formula (original form)

Assumption : B as constant, where

$$B = 0.10\ \frac{d_{50}}{D_*^{\,0.3}}\ \frac{1}{C}\sqrt{\frac{g}{\theta}} \qquad (4.22)$$

$$n = 1 + \frac{3}{1 - \frac{\theta_{cr}}{\theta}\left(\frac{C'}{C}\right)^2} \qquad (4.23)$$

b. Van Rijn (1984): Bed load formula (simplified form)

Assumption: B is constant, where

$$B = 0.005\ d_{50}^{1.2}\ \frac{(C^2 I)^{0.2}}{\left(\sqrt{\frac{g}{C^2\theta}}\right)^{2.4}} \qquad (4.24)$$

$$n = 0.6 + \frac{2.4}{\frac{C\sqrt{\Delta d_{50}\theta}}{u_{cr}} - 1} \qquad (4.25)$$

c. Meyer-Peter Mueller (1948): Bed load formula

$$n = \frac{3}{1 - \frac{0.047}{\mu\theta}} \qquad (4.26)$$

d. Parker and Klingeman (1983): Bed load formula

Assumption: B is constant, where

$$B = \frac{\alpha_2 \, g^{1/2}}{\Delta C^3} \tag{4.27}$$

$$n = 3 \left(\frac{\phi_{50} + 1.644}{\phi_{50} - 0.822}\right) \quad For \ \phi_{50} > 1.65 \tag{4.28}$$

in which:

$$\phi_{50} = \frac{\tau^*_{50}}{\tau^*_{r50}} = \frac{\frac{u^2}{C^2 \Delta d_{50}}}{0.0876} \tag{4.29}$$

4.7 Interpretation of the results

The ranges of 'n' for the selected prediction formulae has been compared against the data shown in Table 3.4.

In the computation of 'n', the median particle size (d_{50}) was taken at 0.22 mm. The Chezy roughness coefficient (C) and the water surface slope (I) were calculated by the following stepwise procedure together with the Engelund-Hansen prediction formula:

(a) assume h'
(b) compute u_*' from the following expression:

$$u = 2.5 \, u_*' \, \ln\left(\frac{12h'}{2.5d_{50}}\right) \tag{4.30}$$

(c) compute θ'
(d) determine θ from the following expressions:

$$\theta' = 0.06 + 0.4 \, \theta^2 \ for \ \theta \le 0.7 \tag{4.31}$$

$$\theta' = \theta \ for \ 0.7 < \theta < 1 \tag{4.32}$$

$$\theta' = (0.3 + 0.7 \, \theta^{-1.8})^{-0.56} \ for \ \theta \ge 1 \tag{4.33}$$

(e) compute h' = (θ'/θ) h and compare with the initial value
(f) repeat until h' = constant
(g) compute C from the following expression:

$$C = 2.5\ g^{0.5}\left(\frac{h'}{h}\right)^{0.5} \ln\left(\frac{12h'}{2.5d_{50}}\right) \qquad (4.34)$$

(h) compute I from the following equation:

$$u = C\sqrt{hI} \qquad (4.35)$$

For each total load prediction formula, the values of 'n' were plotted against the Shields parameter (θ) as shown in Figure 4.5. In this figure, it is seen that most of the formula show an increase of 'n' with a decrease of the Shields parameter. One exception is seen for the Bagnold formula, where 'n' is decreasing with a decrease of the Shields parameter. For the Colby and Engelund-Hansen formulae, 'n' is found to be constant, which is because a term related to the initiation of motion is not included in these formulae. It is also evident from Figure 4.5 that at higher values of θ, 'n' is almost constant, while, at smaller values, the variation is significant. The ranges of 'n' for each formula are shown in Table 4.1.

Predictors	Type of formula	Ranges of exponent 'n'
Ackers and White (1973)	Total load	4.4-6.8
Bagnold (1966)	Total load	3.4-3.9
Engelund-Hansen (1967)	Total load	5.00
Colby (1957)	Total load	3.10
Van Rijn (1984), original form	Total load	3.2-8.5
Yang (1973)	Total load	4.3-5.2

Table 4.1: Ranges of exponent 'n' obtained from total load formulae

For the selected bed load prediction formulae, the values of 'n' were plotted against the Shields parameter (θ), as shown in Figure 4.6. This figure indicates that 'n' computed by the simplified form of the van Rijn formula gives a very much higher range 'n' as compared to the van Rijn original form. The reason could be that the simplified form of the formula was based on computer computations and regression analysis (van Rijn 1992). The range of 'n' for the Parker and Klingeman formula (Parker and Klingeman, 1982) resembles the original formula of van Rijn, whereas the Meyer-Peter and Mueller formula gives smaller values than the original van Rijn formula. The ranges of 'n' for each bed load prediction formula are shown in Table 4.2.

Predictors	Type of formula	Exponent 'n'
Meyer-Peter and Mueller (1948)	Bed Load	3.3-8.6
Parker and Klingeman (1983)	Bed Load	3.7-10.4
Van Rijn (1984) - original form	Bed Load	4.3-10.5
Van Rijn (1984) - simplified form	Bed Load	4.3-29

Table 4.2: Ranges of exponent 'n' obtained from bed load formulae

4.8 Sensitivity analysis

The sensitivity of the exponent 'n' for the selected prediction formulae was compared utilizing the data shown in Table 4.1. This sensitivity analysis was based on variation of two parameters: The grain size (d_{50}), and the water surface slope (I).

The range of 'n' was plotted against the Shields parameter (θ) for different grain sizes and for each total load prediction formula, as shown in Figures 4.7 to 4.10. From these figures, it is seen that the value of 'n' obtained from the Ackers and White and the van Rijn formulae are more sensitive than it is the case for the other formulae used. A finer bed material results in a higher range of 'n', whereas coarser bed material gives a lower range of 'n'.

Also, values of 'n' obtained with different water surface slopes (I), illustrating average conditions of the Jamuna River, were plotted against the Shields parameter (θ), as shown in Figure 4.11. In this figure, it is observed that 'n' is sensitive to the slope for the van Rijn formula at low values of the Shields parameter, but less sensitive for the other formulae used in this study.

4.9 Conclusions

- The exponent 'n' varies between 3.1 to 6 for the general flow conditions of the river, and around 11 for bed load transport. Higher values have been found at the initiation of motion of sediment transport, and in the transition period
- Most sediment transport predictors show an increase of 'n' with a decrease of the Shields parameter (Θ)
- At higher depths, the variation of 'n' is negligible
- It is difficult to derive 'n' at the initiation of motion for the Bagnold formula, because this formula is not derived for that condition

The influence of the slope on 'n' is less significant at lower values of the Shields parameter (Θ)

- The influence of the grain size on 'n' is significant for the Ackers & White and for the van Rijn formulae

5 A Suggested Sediment Transport Equation

5.1 Introduction

As mentioned earlier, an equation representing the relation between sediment transport and discharge is widely used in Bangladesh for estimation of the annual sediment transport and sediment balance. However, this relation has certain limitations, which may be summarised as follows:

- it is a spurious correlation
- it does not explain the scatter in the sediment transport data
- it does not incorporate the hydrodynamic parameters of a river
- it is different from the available prediction formulae

The relation between the Dimensionless Sediment Transport and the Dimensionless Shields Parameter gives a better understanding of the behaviour of the river. Notably, this relation considers the bed shear stress, which is an important variable for sediment transport.

Therefore, a dimensionless sediment transport formula has been developed, utilizing the data collected by BWDB. An inventory of the applied data is given in Table 5.1. In total, 582 sets of data for Jamuna River at Bahadurabad, 196 sets of data for Ganges River at Hardinge Bridge, and 541 sets of data for Padma River at Baruria were used in the analysis. Each set of data represents the sediment transport in a vertical of a measuring transect.

Name of River	Gauging Station	Sediment Transport Data Period (BWDB)	Grain Size of Bed Material River Survey Project FAP 24							Water Level slope Overall 1993-1994
			No of Sample	Collection Period	D_{16} mm	D_{35} mm	D_{50} mm	D_{84} mm	D_{90} mm	
Jamuna	Bahadurabad	1984-1987 (582 sets)	56	1993-1994	0.130	0.160	0.220		0.340	0.000075
Ganges	Harding Bridge	1990-1994 (196 sets)	50	1993-1994	0.100	0.120	0.150	0.180	0.210	0.000055
Padma	Baruria	1986-1987. 1989-1992. 1994 (541 sets)	30	1993-1994	0.100	0.124	0.140	0.185	0.220	0.000040

Table 5.1: Inventory of applied data

The developed sediment transport formula has been compared with the available prediction formulae and has been verified against BWDB data from 1993-1994.

5.2 Dimensionless sediment transport

Most of the existing sediment transport prediction formulae can easily be transformed into a dimensionless form. In this form, the formulae can describe the acceleration and deceleration mode of transport, but cannot explain sediment plumes, i.e. avalanches of bulks of sediment due to river bank erosion, or river bed erosion in the main stream of the river.

Dimensionless sediment transport data from Bahadurabad for the monsoon periods of 1983-1987 are shown in Figure 5.1. From this figure, it can be seen that the samples 195, 196, 200, 202, and 203 are outliers, i.e. behaving differently from the cloud of the sediment samples. An examination of the position of these outliers shows that the sampling verticals were located on high eroding chars (Figure 5.2).
All the sediment samples from Bahadurabad were plotted against their Chezy roughness coefficients in a dimensionless graph (Figure 5.3). The higher roughness values represent samples in the upper layer of the sediment cloud, while the lower values are from the lower layer. There are distinct bundles of sediment samples from distinct roughness groups, which is an indication of the accuracy of the sediment transport measurements. Further, it explains that, at a given Shields parameter, the sediment transport will be higher if the Chezy roughness coefficient increases, i.e. if the bed of the river becomes smoother. The analysis shows that the sediment measurements at Bahadurabad are reliable. Similar analyses were made for the gauging stations Hardinge Bridge and Baruria, as shown in Figure 5.4 and Figure 5.5, respectively. It was found that the roughness bundles are fairly distinct at Hardinge Bridge, but not evident at Baruria. The irregular roughness and scatter in the data could be explained by the constriction effect at Hardinge Bridge, and the confluence effect at Baruria. At these gauging stations, the local hydraulic and morphological parameters are influencing the mode of sediment transport.

The selected prediction formulae in their dimensionless form have been compared with the measured sediment transport, as shown in Figure 5.6. The Bagnold, Engelund-Hansen and van Rijn formulae fit well with the measured regression line. The Bagnold and van Rijn estimates are particularly close to the measured values, whereas the Engelund-Hansen formula needs a little alignment to reach the regression line. Also, the Ackers and White prediction formula is fairly close to the measured line, while estimates according to Yang are lower than the measured values.

The Engelund-Hansen prediction formula has been adopted for modification to fit the measured line because of the simplicity of its equation, which reads as:

$$\psi = 0.05 \frac{C^2}{g} \theta^{2.5} \qquad (5.1)$$

By shifting and rotating this equation towards the measured line in Figure 5.7, the modified Engelund-Hansen formula will read as:

$$\psi = 0.036 \frac{C^2}{g} \theta^{1.83} \qquad (5.2)$$

Equation (5.2) is the modified Engelund-Hansen formula for the Jamuna River.

5.3 Derivation of a Suggested Sediment Transport Equation

The measured suspended sediment transport per unit width (s) is calculated from the measured sediment concentration, the depth averaged flow velocity, and the depth of the vertical. Also, the suspended sediment transport per unit width was calculated for each of the selected prediction formulae. Table 3.2 shows the grain size and the water level slopes used for these calculations.

A linear regression analysis was made of the measured data with the dimensionless Shields parameter (θ) as an independent variable and the dimensionless sediment transport (ψ) as a dependent variable. Hereby, the correlation coefficient was 0.80. A power type equation has been developed relating ψ and θ by applying a regression technique. The relationship obtained is shown in Figure 5.7, and can be expressed as:

$$\psi = 11\ \theta^{1.83} \tag{5.3}$$

This relation can be generalized as:

$$\psi = K_1\ \theta^{n/2} \tag{5.4}$$

Here, the coefficient K_1 is 11. The power 'n/2', which is the exponent of the Shields parameter (θ), is 1.83, and 'n' is equal to 3.66.

The value of the coefficient K_1 can in turn be related to the roughness parameter (Chezy C) of an alluvial river. Therefore, an attempt has been made to incorporate the roughness parameter into equation (5.4). Considering the dimensionless term $C/g^{0.5}$ as a third variable, the coefficient K_1 will read as follows:

$$K_1 = 0.16\left(\frac{C}{\sqrt{g}}\right)^{1.45} \tag{5.5}$$

By substituting equation (5.5) into equation (5.4), the two dimensionless terms ψ and θ can be correlated, and the following relationship is obtained:

$$\psi = 0.16\ \left(\frac{C}{\sqrt{g}}\right)^{1.45}\ \theta^{1.83} \tag{5.6}$$

Equation (5.6) is the Suggested Schematised Sediment Transport Equation (in its non-dimensional form) developed for the Jamuna River. In order to show the degree of performance of the Suggested Equation (Equation 5.4), it has been compared with regression lines generated for various Chezy's roughness C values, as shown in Figure 5.8.

A similar analysis was carried out for the Ganges River at Hardinge Bridge, and for the Padma River at Baruria. As mentioned earlier, the data are scattered at Baruria, and therefore it was not possible to develop any prediction formula.

The data from Hardinge Bridge are characterised by a fairly reliable quality and by the influence of the constriction on the sediment transport (RSP Special Report 18, 1996). For this gauging station, an equation has been derived from the measured velocities and sediment transport, using over-all slope for the average condition. This equation reads as follows:

$$\psi = 0.28\left(\frac{C}{\sqrt{g}}\right)^{1.7} \theta^{2.5} \qquad (5.7)$$

When plotting equations (5.6) and (5.7), it is evident that the derived prediction formula for Jamuna River is not suited for Ganges River. This is shown in Figure 5.9. However, it is possible that the picture would have been different, if the local slope at Hardinge Bridge had been applied, or if sediment measurements from other locations than Hardinge Bridge had been applied.

5.4 Verification of the Suggested Equation

The Suggested Dimensionless Sediment Transport Equation (5.6) for the Jamuna River was derived on the basis of measured data from the Jamuna from 1984-1987. In order to verify the equation, an independent set of BWDB data from the period 1993-1994 was utilized. The result obtained with this new set of data is plotted against the line of perfect agreement in Figure 5.10, which indicates that the Suggested Equation slightly over-predicts the sediment transport rate. However, a correction factor can be used to adjust this variation. The value of this correction factor is found to be 0.75. The measured and the predicted transport rates, using the Suggested Equation in its corrected form, are compared in Figure 5.11.

The accuracy of the Suggested Equation has been calculated by the following discrepancy ratio:

$$R = \frac{\psi_c}{\psi_m} \qquad (5.8)$$

Where

ψ_c = computed bed material sediment transport (m^2/s)
ψ_m = measured bed material sediment transport (m^2/s)

The mean value R and standard deviation σ are defined as

$$R = \frac{\sum_{i=1}^{j} R_i}{j} \qquad (5.9)$$

and

$$\sigma = \sqrt{\frac{\sum_{i=1}^{j} (R_i - R)^2}{j-1}} \qquad (5.10)$$

Table 5.2 shows that the percentage of data that falls within the 0.75-1.25 and the 0.50-1.50 ranges of the discrepancy ratio band are 49 % and 81 %, respectively.

Sediment transport predictor	mean	Discrepancy Ratio				Standard Deviation	No. of data used
		Percent of data in range					
		0.75-1.25	0.50-1.50	0.25-1.75	0.10-1.90		
Suggested equation	1.17	49	81	93	96	0.51	334

Table 5.2: Verification of the Suggested Equation

5.5 Comparison between the different equations

The results obtained by the Suggested Equation and by the five selected sediment transport equations were compared against the measured transport. The criteria selected for this comparison are (i) the percentage of observations within the selected discrepancy ratio band (the discrepancy ratio being the ratio between the predicted variable and the observed variable), and (ii) the distribution of the predicted values against the line of perfect agreement.

The percentage of observations within the selected discrepancy ratio band is shown in Table 5.3. From this table, it is observed that the portion of data falling within the closest discrepancy ratio band (0.75 to 1.25) is found to be 57 %, 32 % and 30 % for the Suggested Equation, the Bagnold equation (1966), and the Engelund-Hansen (1967) equation, respectively. The distribution of the predicted transport rate against the observed data is shown in Figures 5.12 to 5.16 for each prediction formula. From these figures, it is observed that the Suggested, the Bagnold, and the Engelund-Hansen equations comply better with the measured data. A relatively closer agreement is achieved for the Suggested and the Bagnold equations.

The discrepancy ratio and standard deviation of each predictor for Ganges River are shown in Table 5.4.

Sediment transport predictor	Discrepancy ratio					Standard deviation	No. of data sets
	Mean	Percent of data in range					
		0.5-1.50	0.75-1.25	0.25-1.75	0.10-1.90		
Ackers & White	3.19	20	9	28	33	2.52	582
Bagnold	0.68	75	32	97	99	0.30	582
Engelund-Hansen	1.44	53	30	68	76	0.72	582
Van Rijn	1.31	43	24	65	75	0.92	568
Yang	0.41	32	7	75	95	0.24	582
RSP (suggested)	0.88	94	57	98	98	0.44	582

Table 5.3: Accuracy of the prediction formulae, Jamuna River, Bahadurabad

Sediment transport predictor	Discrepancy ratio					Standard deviation	No. of data sets
	Mean	Percent of data in range					
		0.50-1.50	0.75-1.25	0.25-1.75	0.10-1.90		
Ackers & White	3.83	22	13	31	34	2.98	196
Bagnold	0.56	45	12	92	99	0.39	196
Engelund-Hansen	0.79	73	46	96	98	0.39	196
Van Rijn	4.50	21	12	37	44	4.81	196
Yang	0.23	5	1	47	86	0.13	196

Table 5.4: Accuracy of the prediction formulae, Ganges River, Hardinge Bridge

5.6 Comments and recommendation

The analysis of the data from Bahadurabad shows, as a distinct feature, the increase in sediment transport with the increase in Chezy roughness coefficient at a particular dimensionless Shields parameter. This indicates that the data are reliable, both with respect to the site of the gauging station and with respect to the measurements as such.

Based on the 582 sets of data from each vertical at Bahadurabad, the exponent 'n' of the Suggested Equation has been found to be 3.70. This value lies within the range derived from the theoretical analysis.

A comparison of the Suggested and the selected prediction formulae shows an accuracy in the following descending order: Suggested, Bagnold, Engelund-Hansen, Yang, van Rijn, and Ackers and White.

The analyses of data from Hardinge Bridge and Baruria show a fair and a poor quality of the measurements, respectively. Here, possibly, the morphology at the location of the gauging stations are influencing the data quality:

- A constriction effect at the Hardinge Bridge gauging station may be the cause of the observed data scatter and inaccuracy. Using the local slope instead of the over-all slope in the analysis could have improved the data quality

- The Jamuna/Ganges confluence a few km upstream of the Baruria gauging station may influence the mode of sediment transport at Baruria. Also here, it is possible that an improvement can be obtained by introducing the local slope in the analysis, rather than the over-all slope, and by shifting the gauging station to a more suitable location

No data are available on local slope at these gauging stations, so it is difficult to identify the reasons for the data inconsistencies. However, mathematical modelling is a tool which has the flexibility required to explore the issue, qualitatively if not quantitatively. This is because a mathematical model can generate the local slope at any required location, for comparison with the over-all slope. Therefore, as described in the following chapter, a 1-dimensional mathematical model has been applied for explaining the effect of the constriction and the confluence at the respective gauging stations.

6 1-dimensional mathematical modelling

6.1 Introduction

1-dimensional morphological modelling is one of the main tools available for predicting the long-and medium-term morphological response of an extensive river system. In the RSP, the Ganges-Jamuna-Padma Model (the GJP Model) has been developed for (at least) a qualitative examination of the inconsistencies observed in connection with the dimensionless rating curves from the field data from Bahadurabad, Hardinge Bridge, and Baruria.

The model is based on a schematization of the physical system subject to certain restrictive assumptions. Therefore, the results must be interpreted with a good understanding of the physical processes, and a proper awareness of the assumptions made during the schematization.

Although actual measured data are used for the simulations, the interpretation of the results is qualitative rather than quantitative. Still, there is no doubt that the model can support a deeper understanding of the sediment transport and the morphological phenomena, and hereby give a clue to the inconsistencies noticed by the RSP during the data analysis.

Some studies of different authors were summarised in Chapter 3, including findings and theories that relate to the inconsistencies. In this Chapter, the matter is examined further by the GJP Model.

6.2 Schematization of river channels

The schematization of the river network of the GJP Model is shown in Figure 6.1. The principal features of the model are as follows:

1. The cross-section schematization was made with a rectangular profile as follows (see Figure 6.2):
   - i Divide the cross-section into i segments
   - ii Compute $Y = \Sigma B_i(h_i)^{1.5}$
   - iii Compute the bed level z of the schematized cross-section:
     $(H_D - Z_b)^{1.5} = [\Sigma B_i (h_i)^{1.5}] / B$ (with $B = \Sigma B_i$)
2. All lateral inflows and outflows of water from these three rivers are assumed to be negligible and have therefore not been included in the model
3. The Engelund-Hansen sediment transport formula is used in the model for simulation of sediment transport
4. A 50 years simulation period was achieved by repeating the 1987 boundary values 50 times, in order to reach a dynamic equilibrium

A schematization like the one applied is not well suited for accurate high and low water simulations. This is because the cross-section flow area is not well represented for very high and very low discharges. For morphological computations, this shortcoming is not very important: During low discharge, the sediment transport is very low, and the morphological changes will be small; and very high discharges have a low frequency of occurrence.

The bed roughness is computed by the White and Colebrook equation, where the Chezy coefficient is a measure of the roughness. The bed material grain sizes used in the model are listed in Table 3.2 (Chapter 3).

The upstream boundaries of the Jamuna and Ganges have been extended further upstream using the same schematized cross sections, in order to delay the propagation of boundary errors into the area of interest.

6.3 Applications

The main objective of the modelling is to explore the cause of the inconsistencies in BWDB's sediment transport data. The following factors have been analyzed:

1. The constriction of the Ganges River at Hardinge Bridge to a width of 1.6 km from the main river width of 3.8 km

2. The Ganges/Jamuna confluence and its effect on the sediment transport and the bed level changes at adjacent reaches

6.4 Study of inconsistencies

Potential inconsistencies in the calculations can be indicated by (i) a scatter of the values, and (ii) a steep dimensionless rating curve.

Constriction effect at the Hardinge Bridge gauging station

The possible cause for scatter and a steep dimensionless rating curve at Hardinge Bridge could be the constriction effect.

In the model, a 1600 m wide and 2000 m long constriction was introduced at the river training reach of Hardinge Bridge on the Ganges River, while a uniform width of 3800 m was applied for the unconstricted river.

Two cases were studied: One 'without constriction' and another one 'with constriction'. The dimensionless sediment transport for both cases are shown in Figure 6.3a. This figure shows that in case of a constriction, the sediment transport is very scattered, whereas, without constriction, the transport is substantially less scattered. The power law dimensionless sediment rating curves are plotted in a logarithmic scale in Figure 6.3b. The dimensionless sediment rating curve is steeper at the constricted section (n/2 = 4.60) than at the unconstricted section (n/2 = 1.87). The slope used in the Shields parameter is the over-all slope computed from water levels from two sections located 80 km apart.

In order to improve the quality of the sediment transport data in their dimensionless form at the constriction, an attempt has been made to verify the above analysis with the introduction of the local slope in the Shields parameter. This local slope is computed over the grid increment that contains the section from where the data are extracted. The data in their dimensionless form are plotted for both the over-all and the local slope in Figure 6.4a. In this figure, it can be seen that the quality of the (dimensionless) data is highly improved (indicated by a reduced scatter), when the local slope is used in the Shields parameter.

The rating curves derived from these data are shown in Figure 6.4b, and the power law dimensionless sediment rating curves are plotted in a logarithmic scale in Figure 6.4c. At the constriction, the dimensionless sediment rating curve is steeper when using the over-all slope in the Shields parameter than when using the local slope (n/2 = 4.6, as compared with n/2 = 3).

The scatter in the data at Hardinge Bridge can be explained by Figure 6.5, which shows that a scour hole has developed at the vicinity of the constriction. This scour hole has a significant local influence on the mode of sediment transport. The figure shows that the water level varies within the scour hole, especially in the lean period. Therefore, the (local) slope within the scour hole is governing the sediment transport, and the use of an over-all slope in the Shields parameter will give erroneous results, as this slope is not influenced by the scour hole. At low to moderate discharges, the sediment transport is reduced by deposition in the scour hole, while, at higher discharges, this deposition is washed away, whereby the sediment transport becomes high. These exceptionally high and exceptionally low sediment transports appear as a scatter.

Confluence effect at the Baruria gauging station

The Baruria gauging station is located downstream of the Jamuna/Ganges confluence. At this station, the sediment transport data are very scattered, so that dimensionless sediment rating curves could not be produced.

For simplicity, a single uniform cross-section for Padma river was used in the model.

The modelling results show a characteristic scatter in the dimensionless sediment transport data at different sections of the Padma River from the confluence and downstream (Figure 6.6a). In this figure, the over-all slope is used in the Shields parameter. Hereby, the slope is estimated from water levels at 5 km and 50 km downstream of the confluence. At the confluence, the scatter is significant, while, further downstream, it is greatly reduced. Downstream of the confluence, the dimensionless sediment transport is moderately low and scattered at a small Shields parameter, but the transport gets abruptly high when the Shields parameter increases above 1 (one). Consequently, a dimensionless sediment rating curve based on all these data will have a progressively steep slope. In reality, there are two slopes: One at low Shield parameters (the sediment transport being negligible), and another at higher Shields parameters.

When the local slope is introduced in the Shields parameter, the dimensionless sediment transport appears as shown in Figure 6.6b. Here, the scatter in the data is greatly reduced for the downstream sections, but it remains high around the confluence. Therefore, in Figure 6.7, the dimensionless sediment rating curves have been drawn for the downstream sections, where the scatter is less. In Figure 6.7a, at a distance of 10.5 km from the confluence, the dimensionless sediment rating curve is steep (n/2 = 7.5) when based on the over-all slope, and less steep (n/2 = 3.5) when based on the local slope. In Figure 6.7b, a similar effect is noticed at a section 15 km downstream of the confluence, where n/2 = 8 with the over-all slope and n/2 = 3.2 with the local slope.

At the confluence (Figure 6.6a), the higher sediment transport during rising stage is a result of a greatly increased supply of sediment from the Jamuna River due to scouring of its bed in the area near to the confluence. The low sediment transport during falling stage is the result of a reduced supply of sediment from upstream branches. The reaches upstream of a confluence are all the time adjusting, but are, at the same time, fluctuating around an average bed level (DELFT/DHI, 1993e), so the system is in a dynamic equilibrium. This is illustrated in Figure 6.8: During rising stage, the river bed is degrading, and during falling stage it is aggrading. This aggrading in the Jamuna is a typical feature of the backwater effect in this river due to the late flood in the Ganges River. Downstream of the confluence, alternating processes of aggradation and degradation occur. This gives rise to generation of sand waves, which travel through the downstream reach (Figure 6.9). These bed level changes in the different branches near the confluence are the main reason for the scatter observed in the Padma River. At the confluence, the annual fluctuation of the bed is high, in the order of 1.5 to 2 m (confluence scour). The fluctuations gradually become insignificant further downstream.

7 Conclusion and recommendation

7.1 Quality of data

The suspended sediment transport measured by BWDB at the Bahadurabad transect distinctly reflects the bed roughness, and the data are not very scattered. An increase in sediment transport is associated with an increase in Chezy roughness at a particular Shields parameter. This indicates that the quality of data at Bahadurabad is reliable with respect to the morphological condition of the gauging station, and also with respect to the measurements as such. Estimates of the sediment transport based on the over-all slope are reliable.

The data at the Hardinge Bridge gauging station are scattered, and it is difficult to produce a dimensionless sediment rating curve. Still, a rating curve was made. It has a steep slope and shows a significant deviation from the rating curve at Bahadurabad. The 1-D modelling shows that the constriction due to the Hardinge Bridge is the main cause for the data discrepancies: In the vicinity of an artificial constriction in a natural river, the constriction causes a lower sediment transport during low to moderate discharge, and a higher transport during high discharges. This increased transport variation between low and high discharges is the reason behind the steep dimensionless rating curve.

The data from Baruria are very scattered. The mathematical modelling shows that the confluence and its reaches are under a continued dynamic morphological adaptation. A historical analysis shows that the Jamuna and the Ganges have different patterns of hydrological events. The former has an earlier flood than the latter, but also the flow variations at low discharges are very uneven in the two rivers. At the confluence, these differences cause an abrupt lowering of the water level in one channel, and backwater effects in the other, which create erosion and sedimentation in the respective channels. All these processes result in an uneven and scattered sediment transport. Also, an analysis has shown that downstream of the confluence, a big natural scour area develops, which further contributes to the scatter in the sediment transport (see RSP Special Report 18, 1996). This analysis also shows that the confluence effect remains for a long distance in the downstream channel.

For the production of dimensionless sediment rating curves at the above mentioned gauging stations, the over-all slope was used in the Shields parameter. The mathematical model shows that introduction of the local slope can significantly improve the data quality at Hardinge Bridge, but not significantly for the Baruria station. However, it is difficult to measure the local slope at both these stations, because the morphology is abruptly changing within a short distance.

In order to obtain sediment data with a high quality, the location of a site of a gauging station should be selected carefully according to several criteria, as explained in this study and in gauging manuals (WMO, 1989). The gauging section should be free from artificially narrowed channels, and should be away from an upstream confluence. At Hardinge Bridge, the contraction scour of the river bed induces a yearly variation in the sediment transport which is not representative for the whole reach of the Ganges River. The sediment transport data from this contraction will not lead to good estimates of the over-all sediment budget. Also, upstream of Hardinge Bridge, there is a bend, which again implies that this station is not well suited for sediment gauging. Therefore, in order to obtain valid data for morphological studies, the location of this gauging station should be changed. A section downstream of the bridge can provide better data. The exact location should be selected after a detailed investigation.

At Baruria, morphological changes of the river bed are predominant over the year. Satellite images show a significant shifting of the confluence. The confluence with its scour has a significant effect on the sediment transport data at Baruria. Therefore, it is recommended to shift this gauging station to a more stable location further downstream. But this shift should be limited as much as possible to reduce the influence of the tidal variations on the measurements.

If an exact calibration of a model can be done, then the model becomes a suitable tool for determination of the local slope at a station where the morphological variation is significant.

7.2 The exponent 'n'

The flow velocity exponent 'n' for alluvial rivers ranges between 3.1 to 8.5. The value of 'n' is considered a dominant parameter of sediment transport in alluvial rivers. The graphical relation between 'n' and the Shields parameter (θ) shows an increase of 'n' with a decrease of θ, which implies that near the initiation of motion, 'n' is expected to be high, as it is the case with most of the prediction formula.

7.3 A prediction formula for Jamuna River

A schematised sediment transport equation, the Suggested Equation, has been developed for the Jamuna River at Bahadurabad. Subsequently, the equation has been verified with independent sets of data from another period, with a satisfactory result.

Based on the 582 sets of data from Jamuna, the over-all accuracy of sediment predictors in descending order are the RSP Suggested Equation, Bagnold, Engelund-Hansen, Yang, van Rijn, and Ackers and White.

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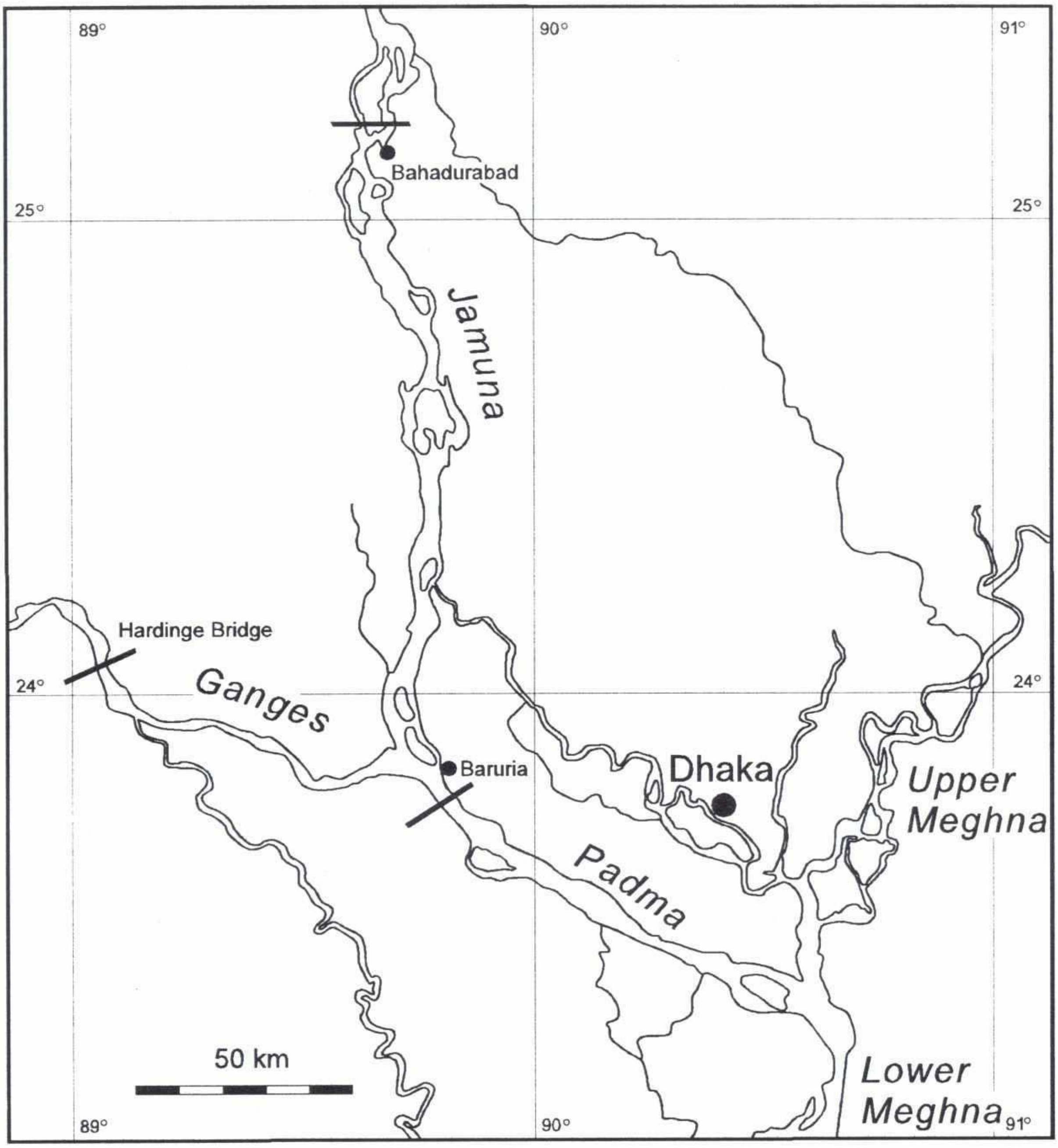


Figure 3.1: Location map

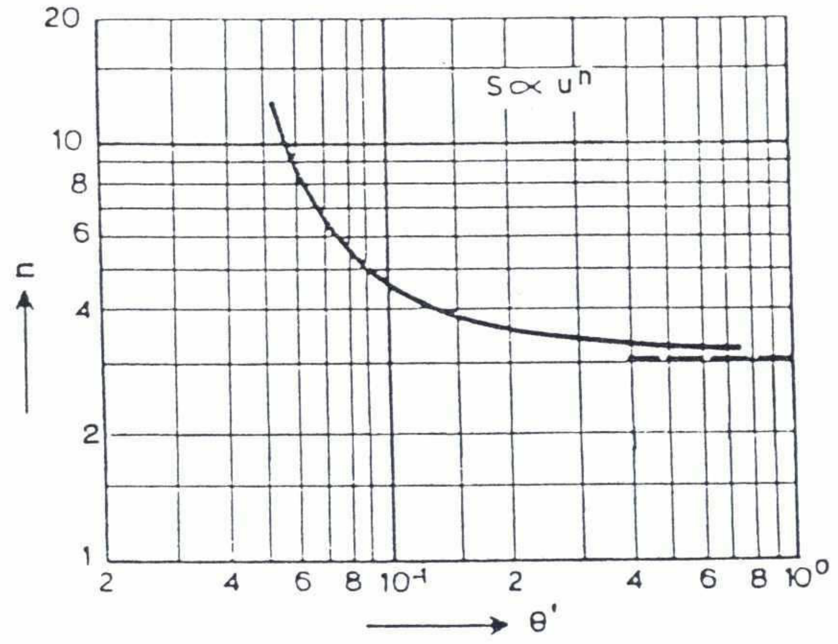


Figure 4.1: Approximate relationship between n and θ'

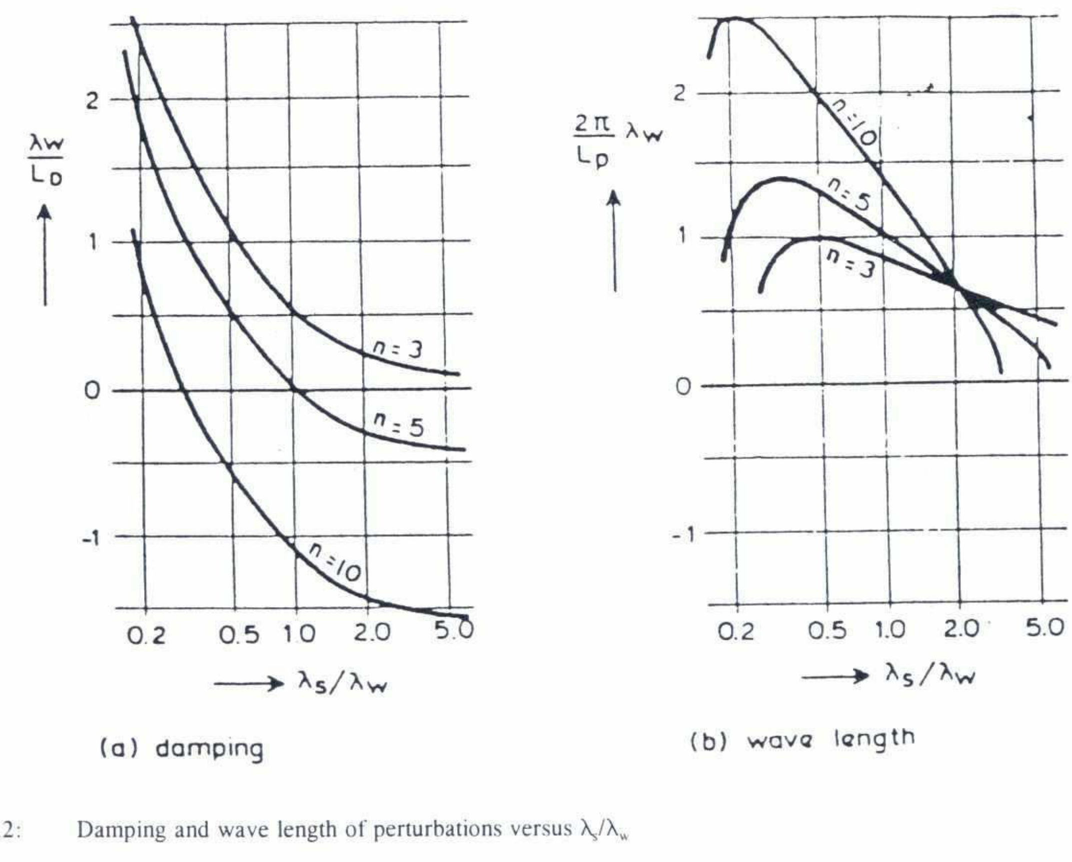


Figure 4.2: Damping and wave length of perturbations versus λ_s/λ_w

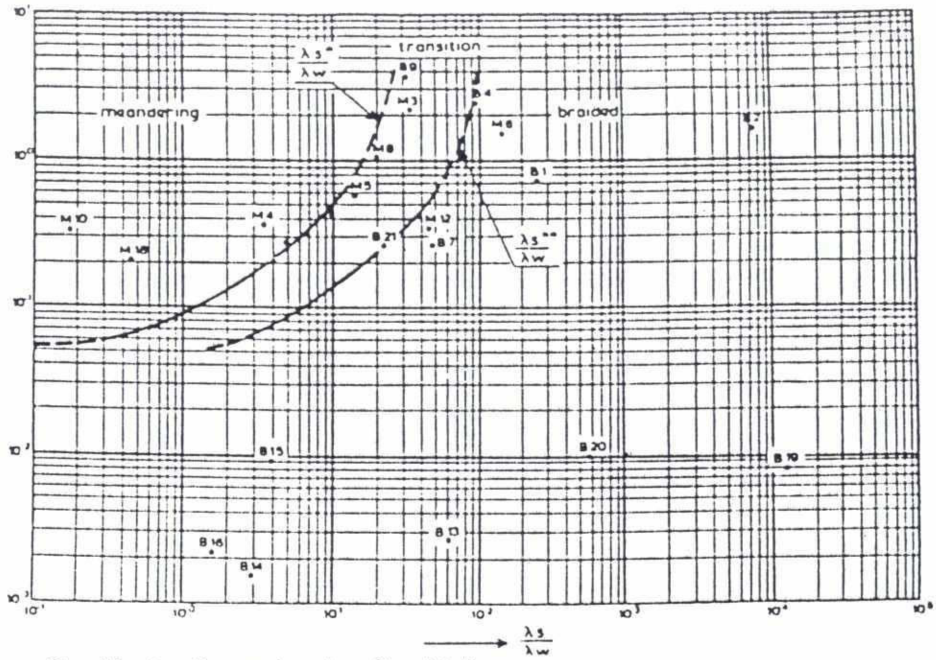


Figure 4.3: Classification diagram based on θ' and λ_s/λ_w

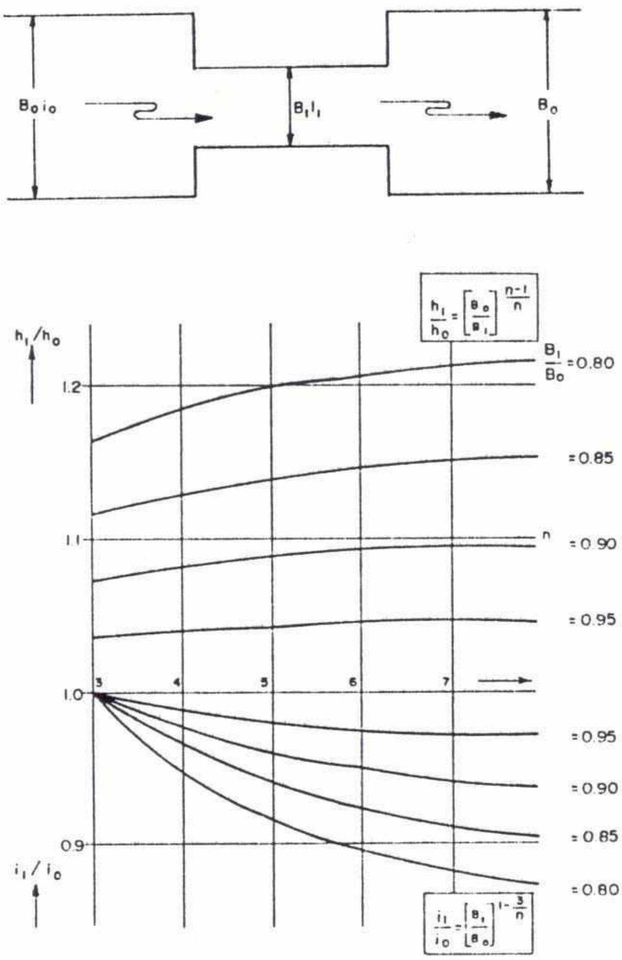


Figure 4.4: Consequences of constriction of river width

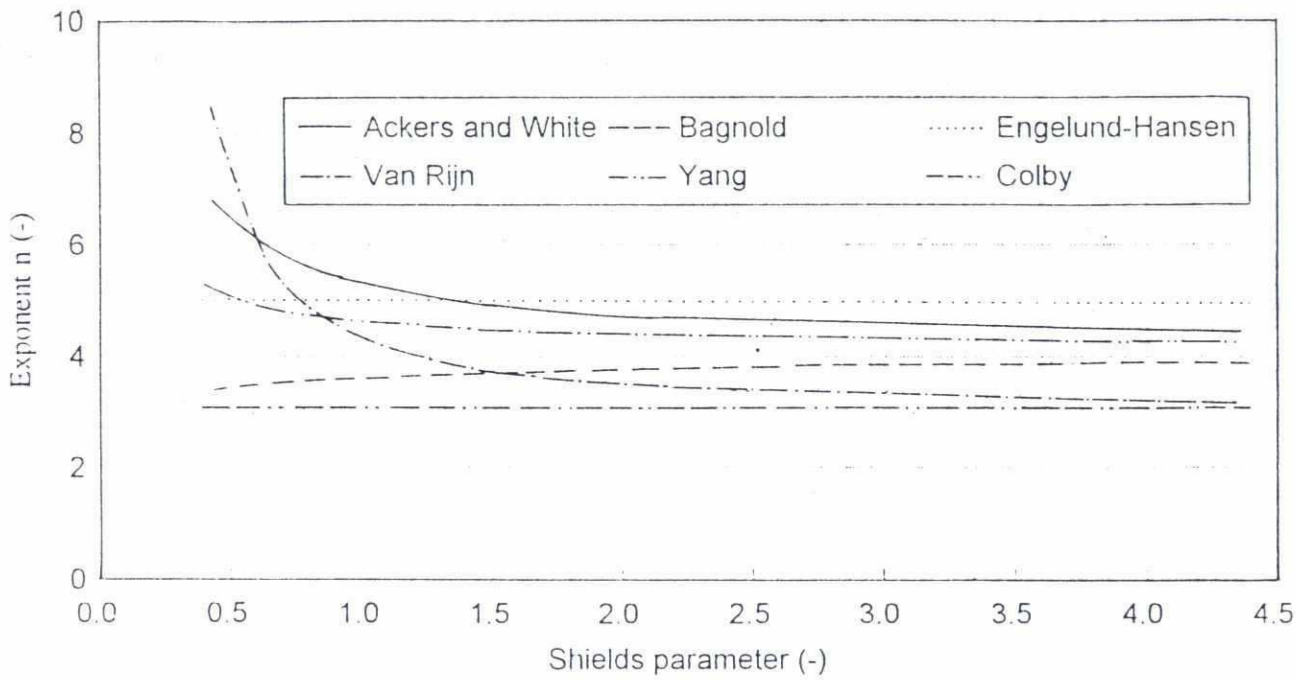


Figure 4.5: Range of exponent 'n' for total load prediction formulae

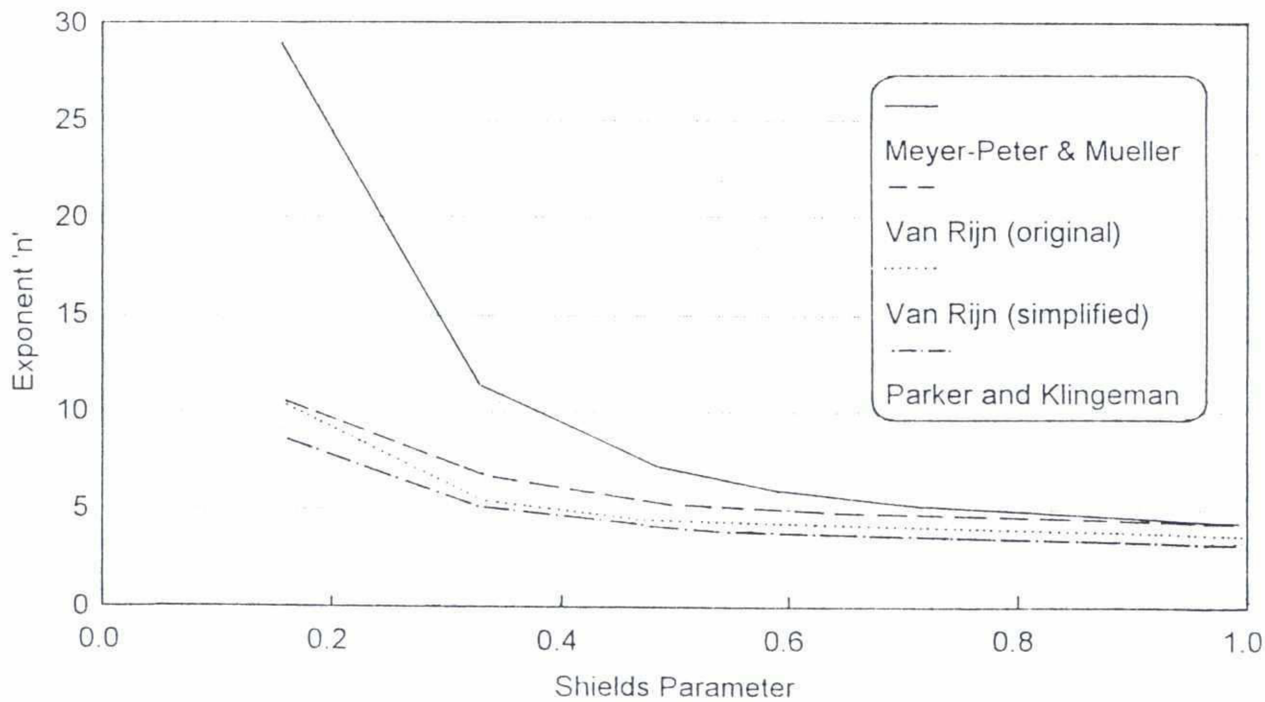


Figure 4.6: Range of exponent 'n' for bed load prediction formulae

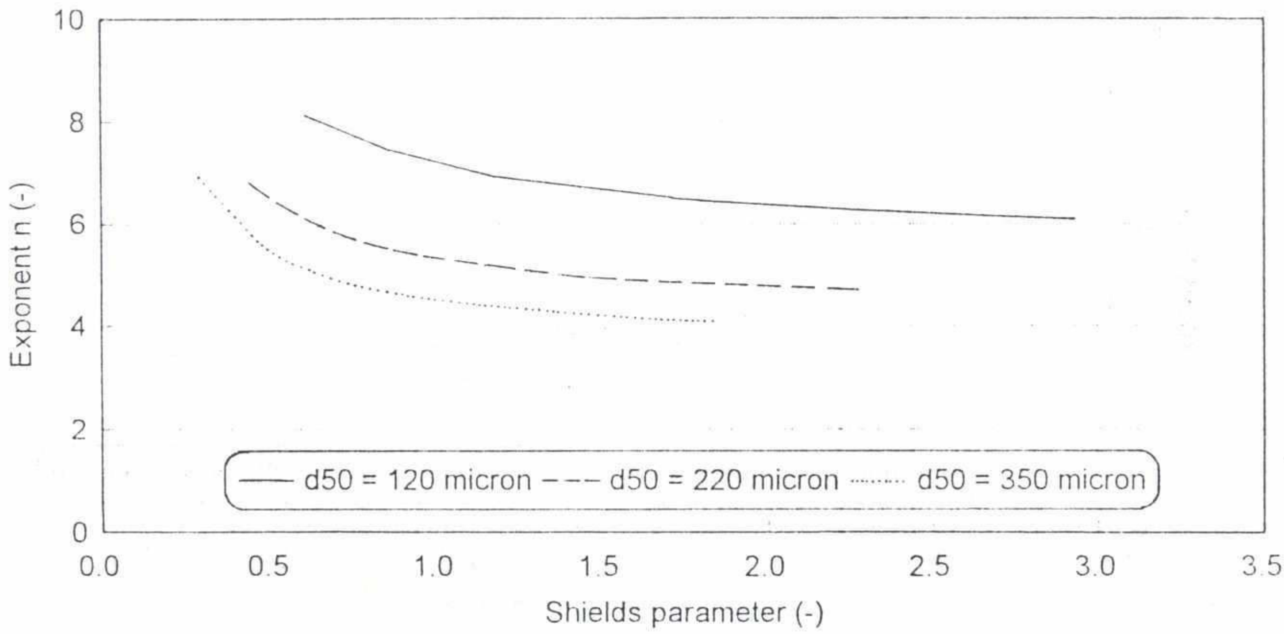


Figure 4.7: Sensitivity of exponent 'n', Ackers & White formula

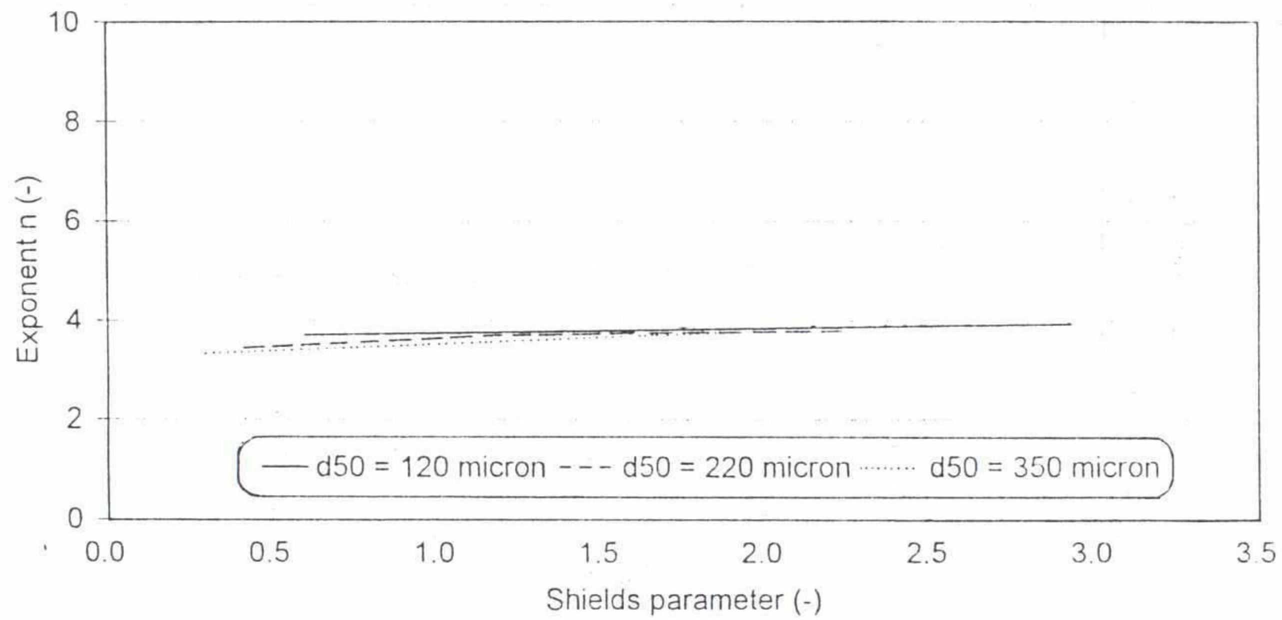


Figure 4.8: Sensitivity of exponent 'n', Bagnold formula

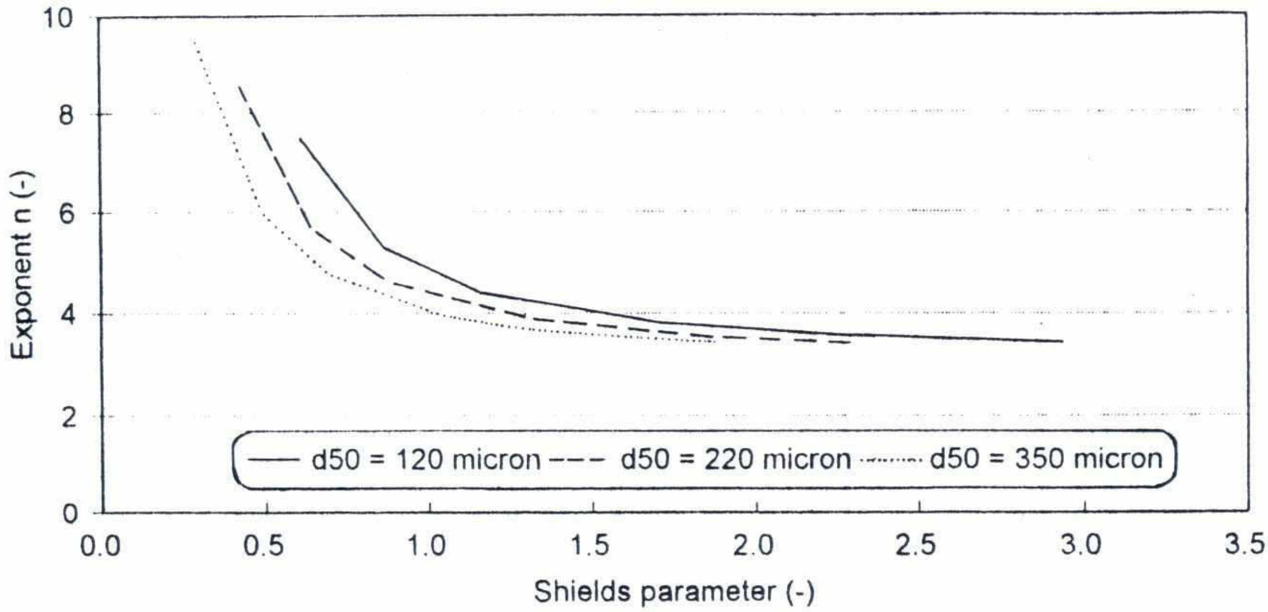


Figure 4.9: Sensitivity of exponent 'n', van Rijn formula

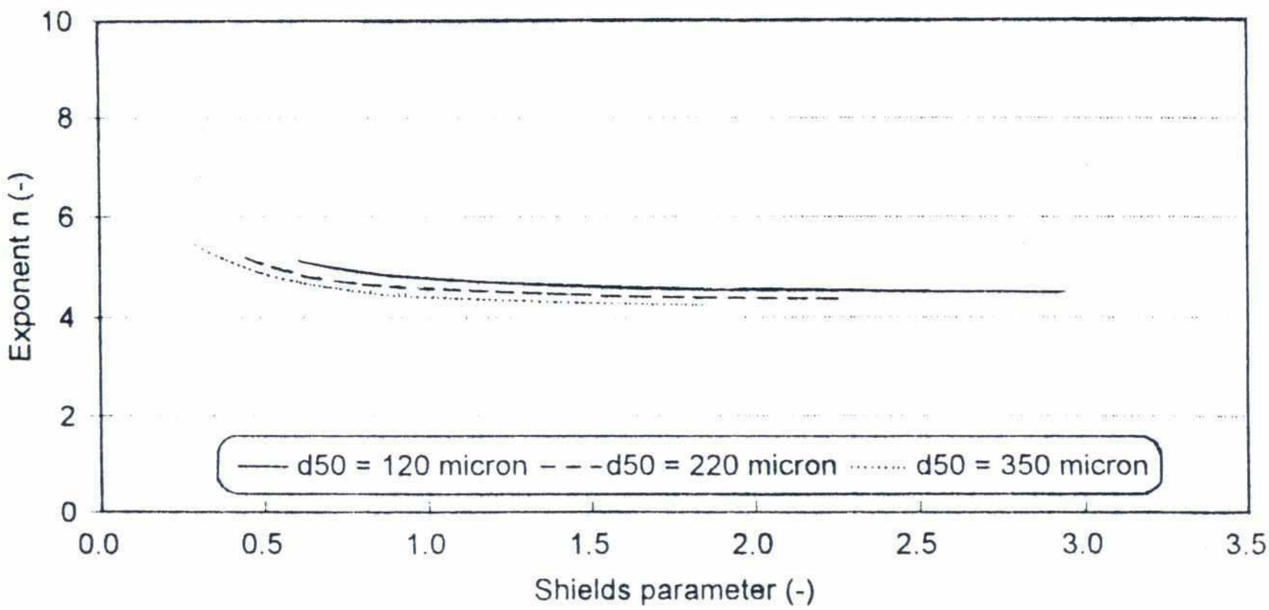


Figure 4.10: Sensitivity of exponent 'n', Yang formula

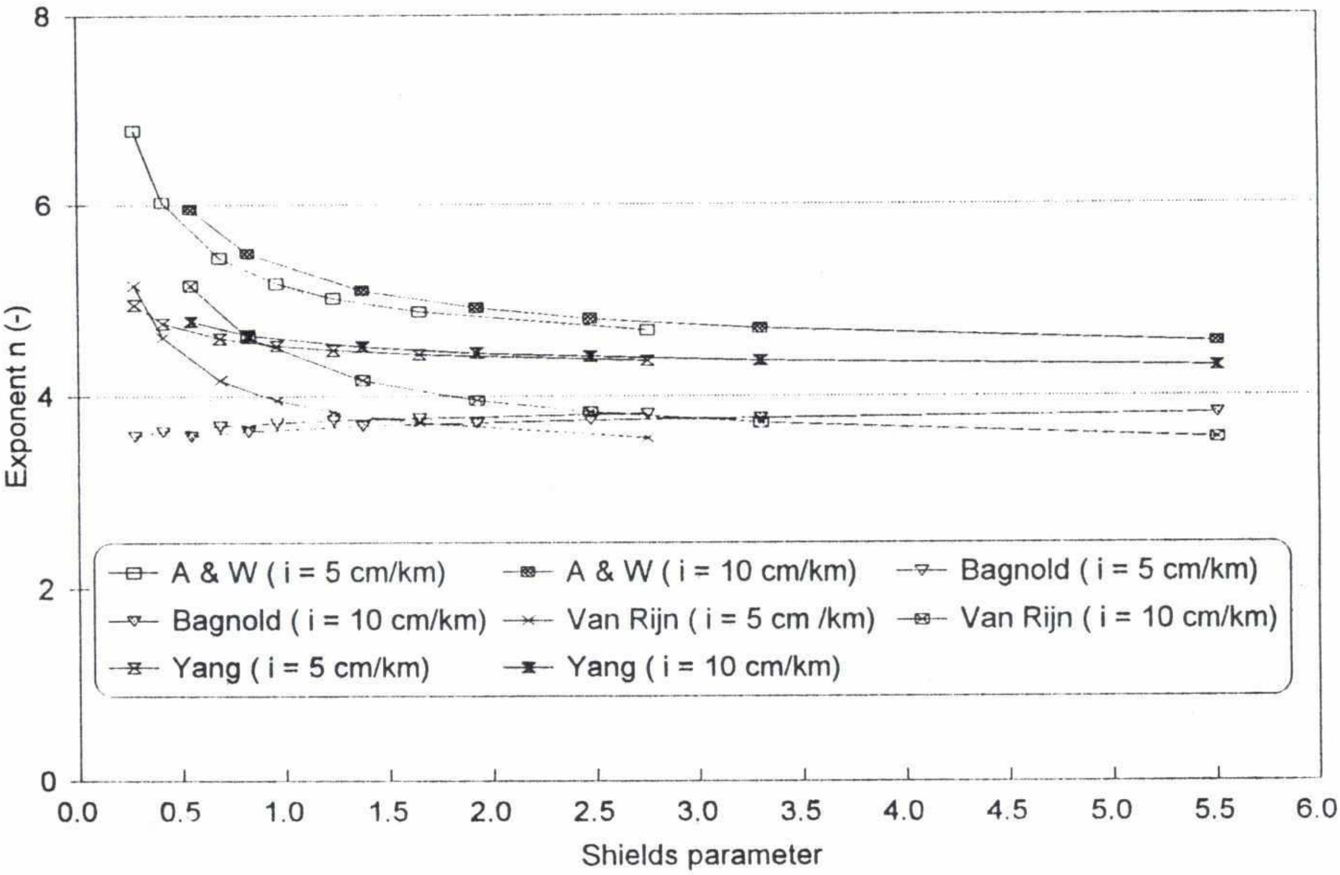


Figure 4.11: Sensitivity of exponent 'n' for variation of slopes, total load formulae

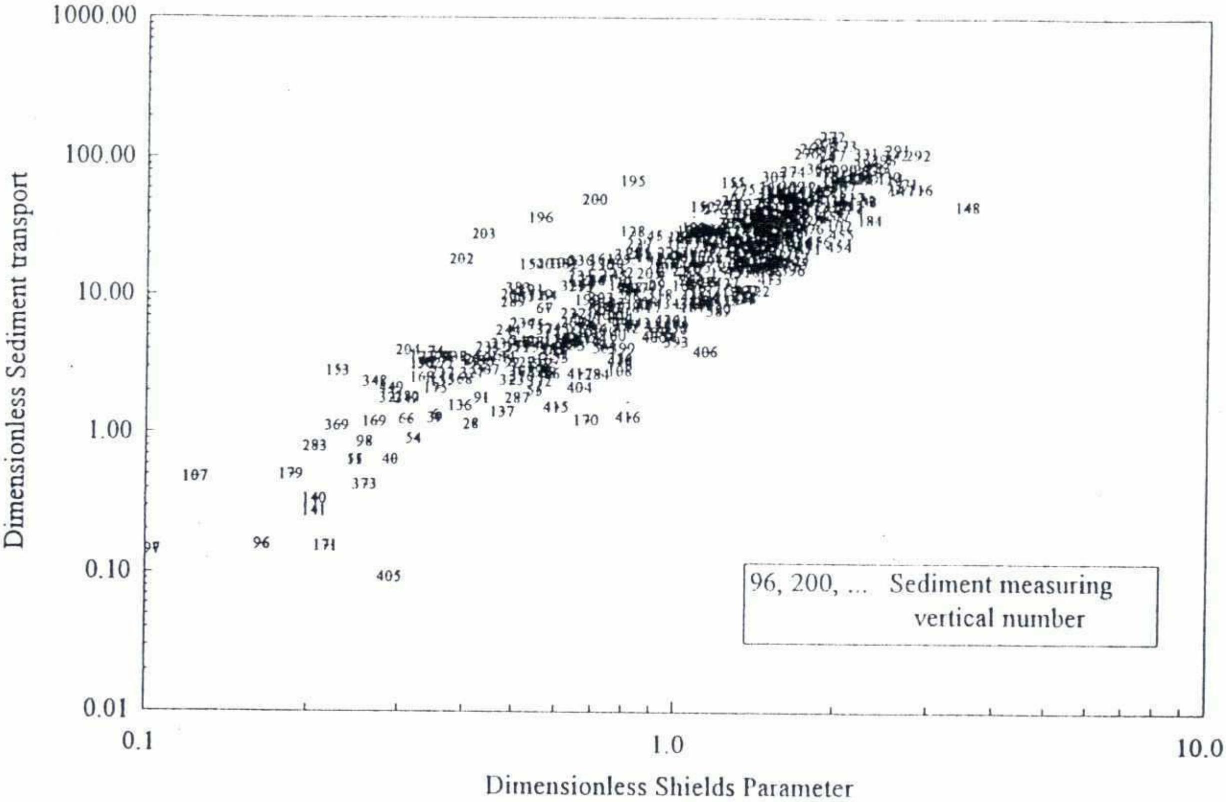


Figure 5.1: Sediment transport data, Bahadurabad, 1983-87, monsoon period

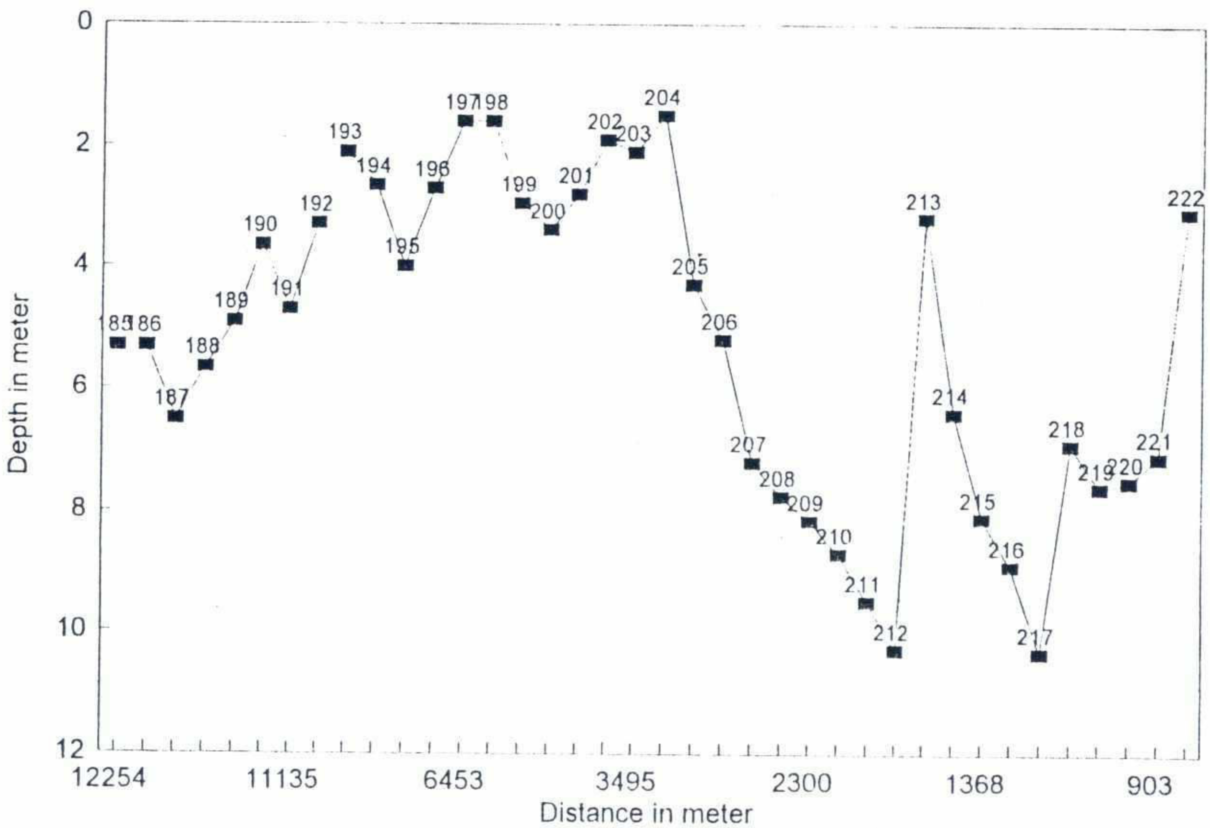


Figure 5.2: Location of verticals, Bahadurabad

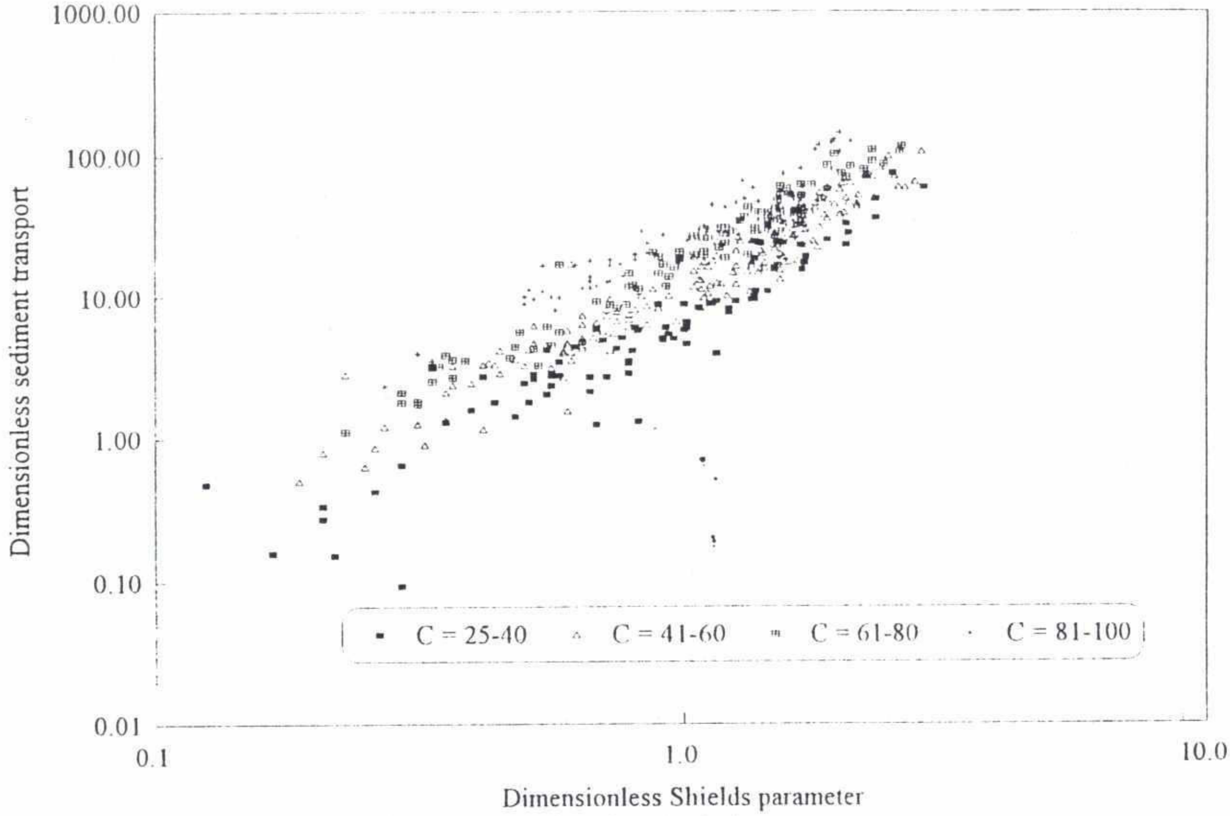


Figure 5.3: Chèzy roughness of sediment transport data, Bahadurabad

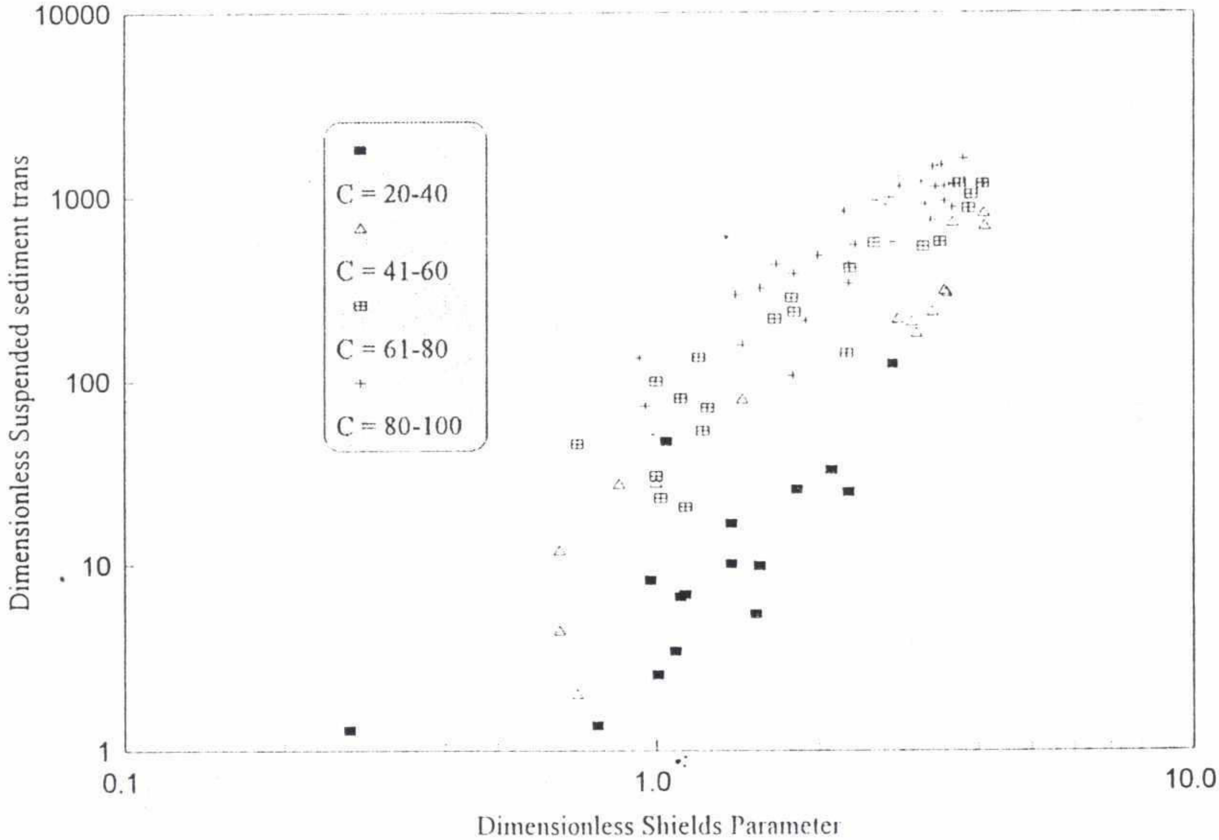


Figure 5.4: Chèzy roughness of sediment transport data, Hardinge Bridge

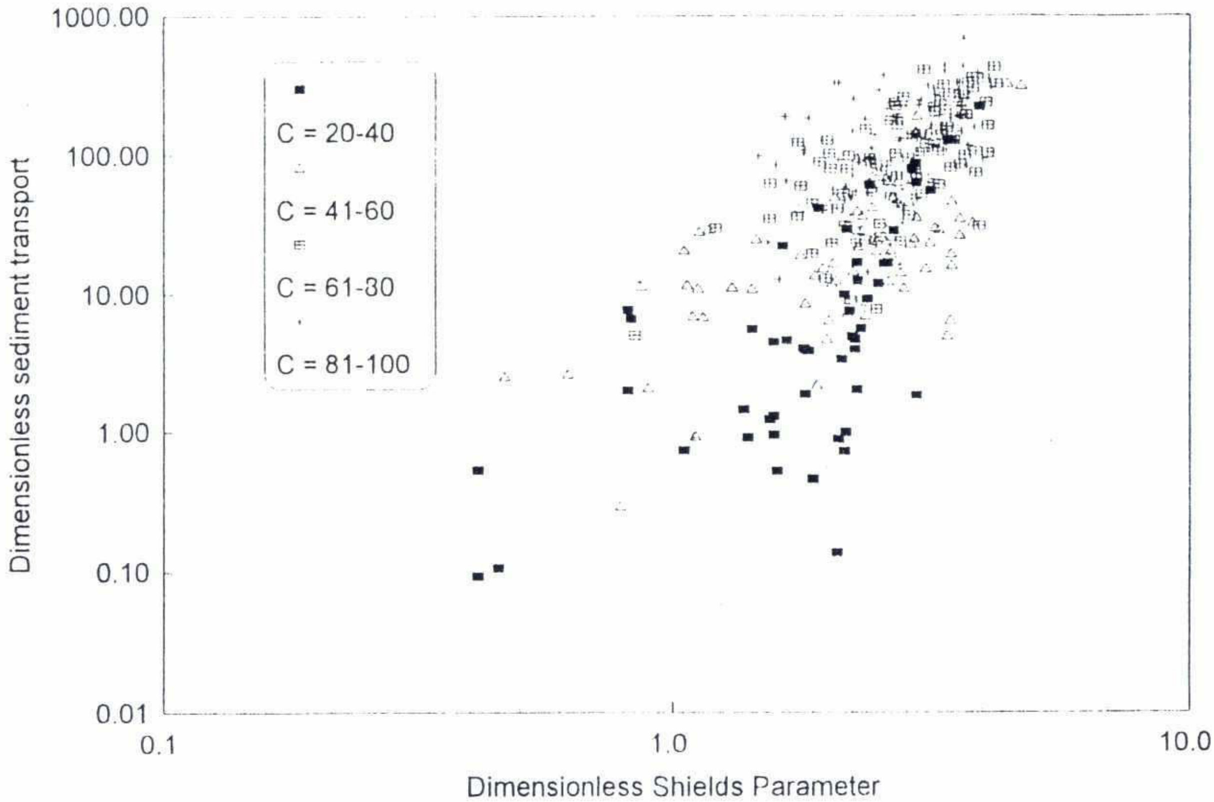


Figure 5.5: Chèzy roughness of sediment transport data, Baruria

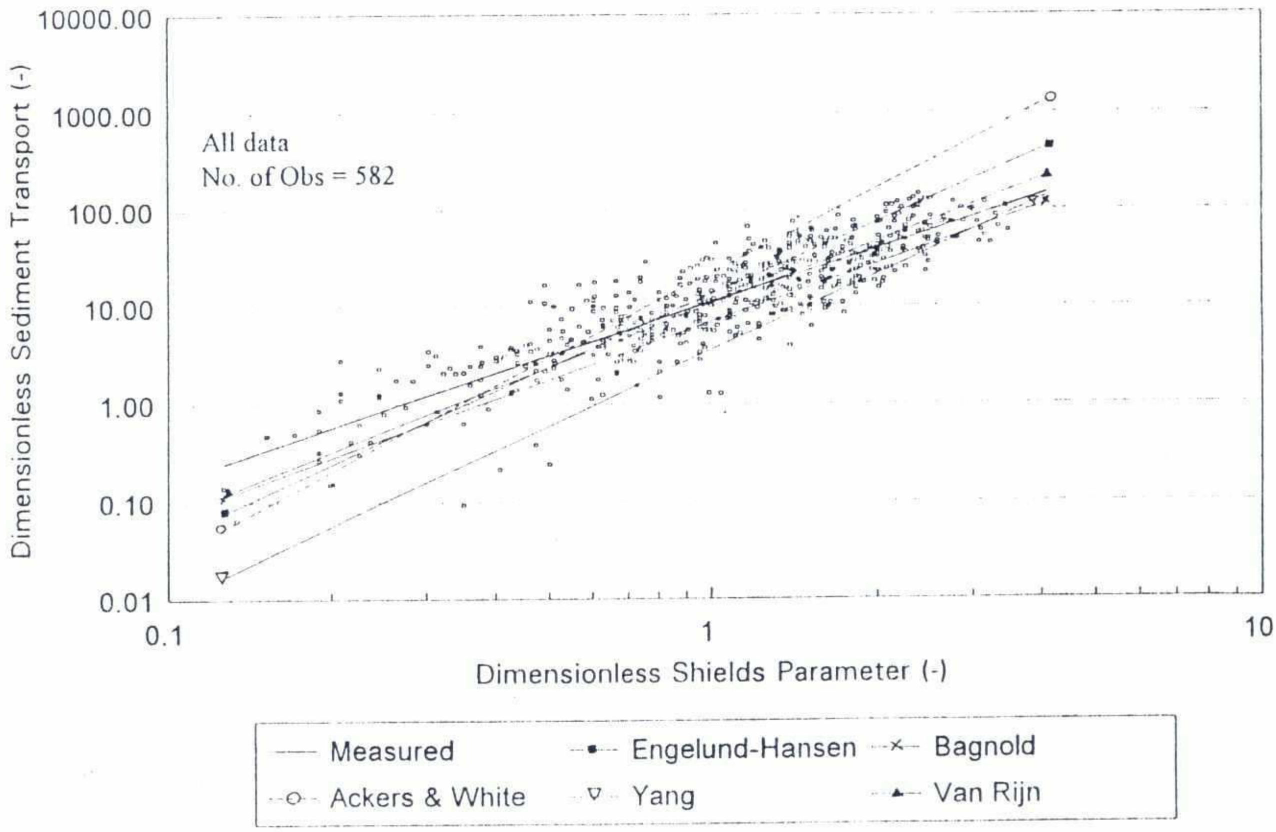


Figure 5.6: Different prediction formulae compared with measurements

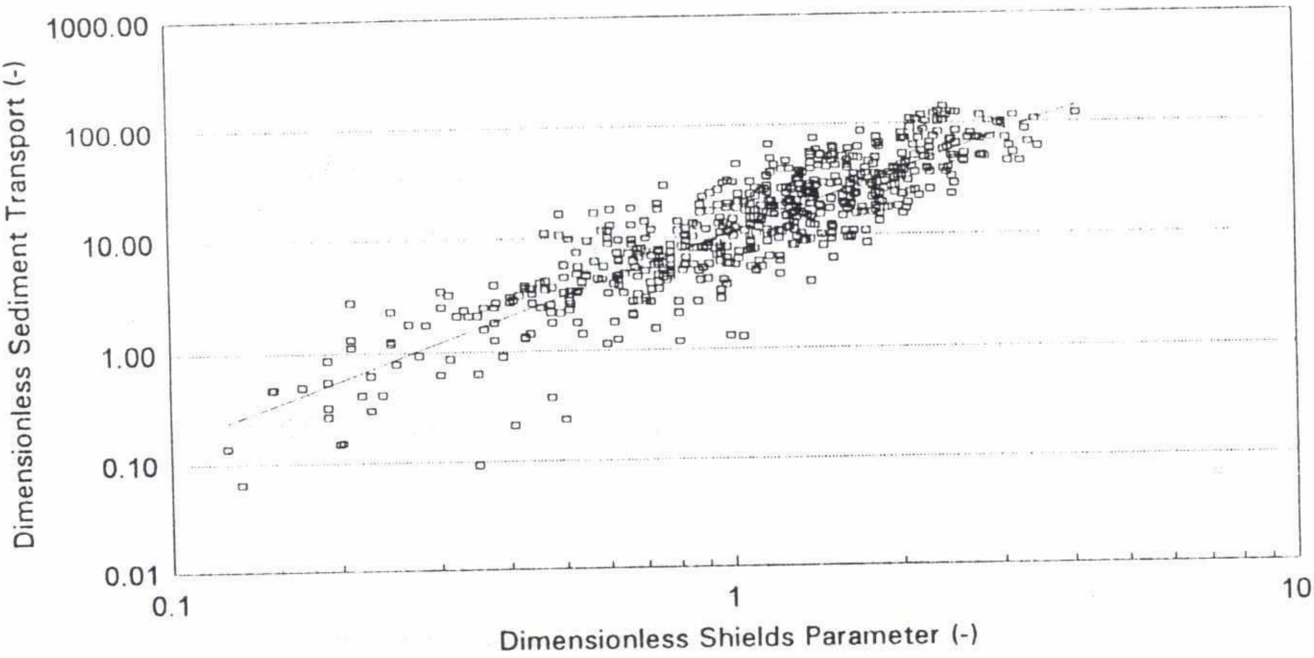


Figure 5.7: Dimensionless sediment rating curve, Jamuna

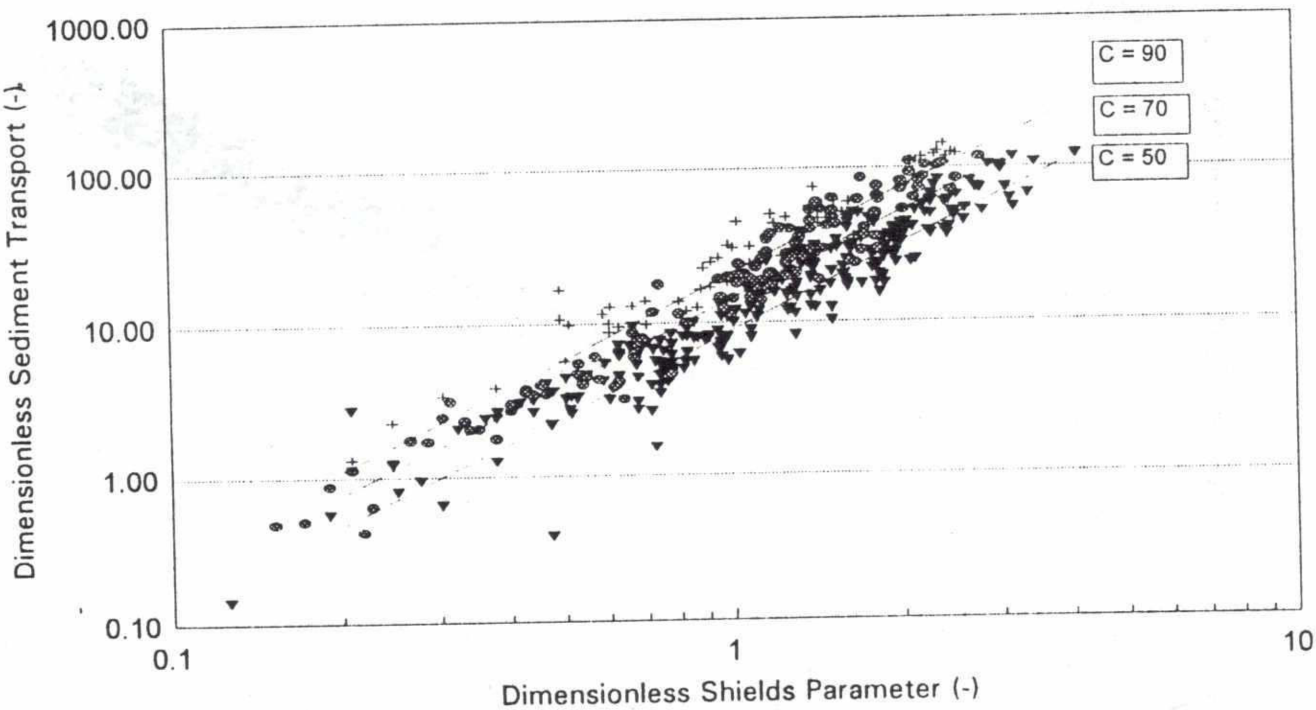


Figure 5.8: Dimensionless sediment rating curves by Chèzy roughness, Jamuna

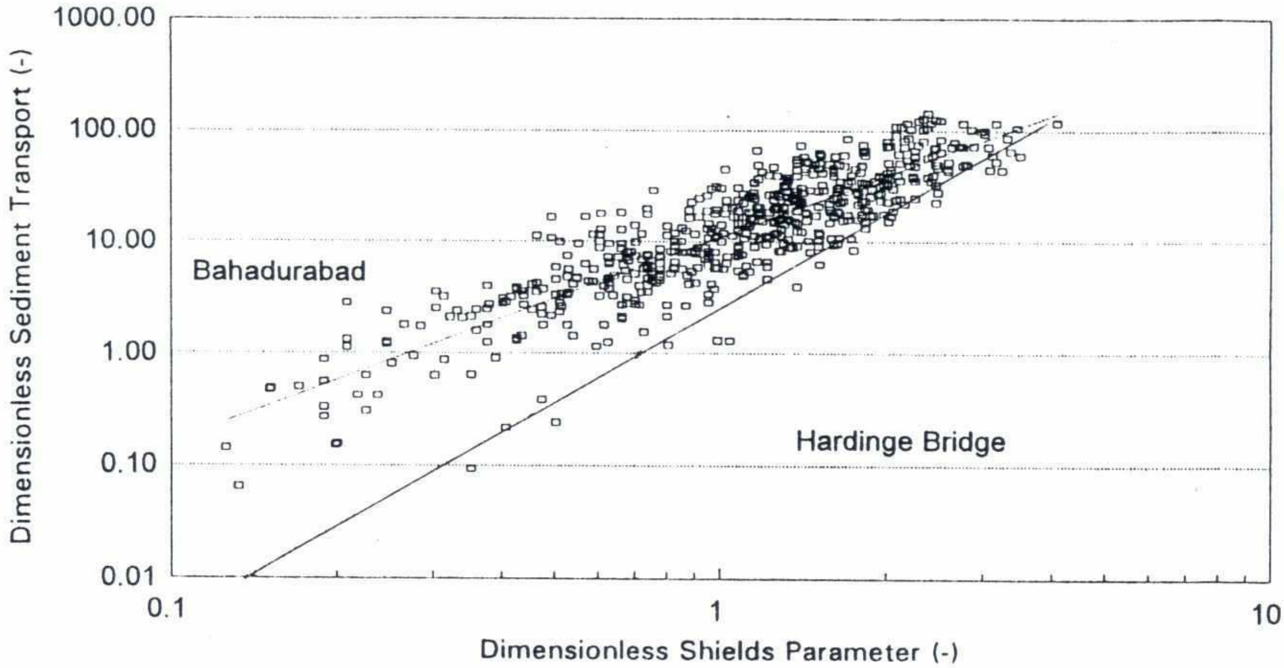


Figure 5.9: Comparison between Suggested Equations, Jamuna and Ganges

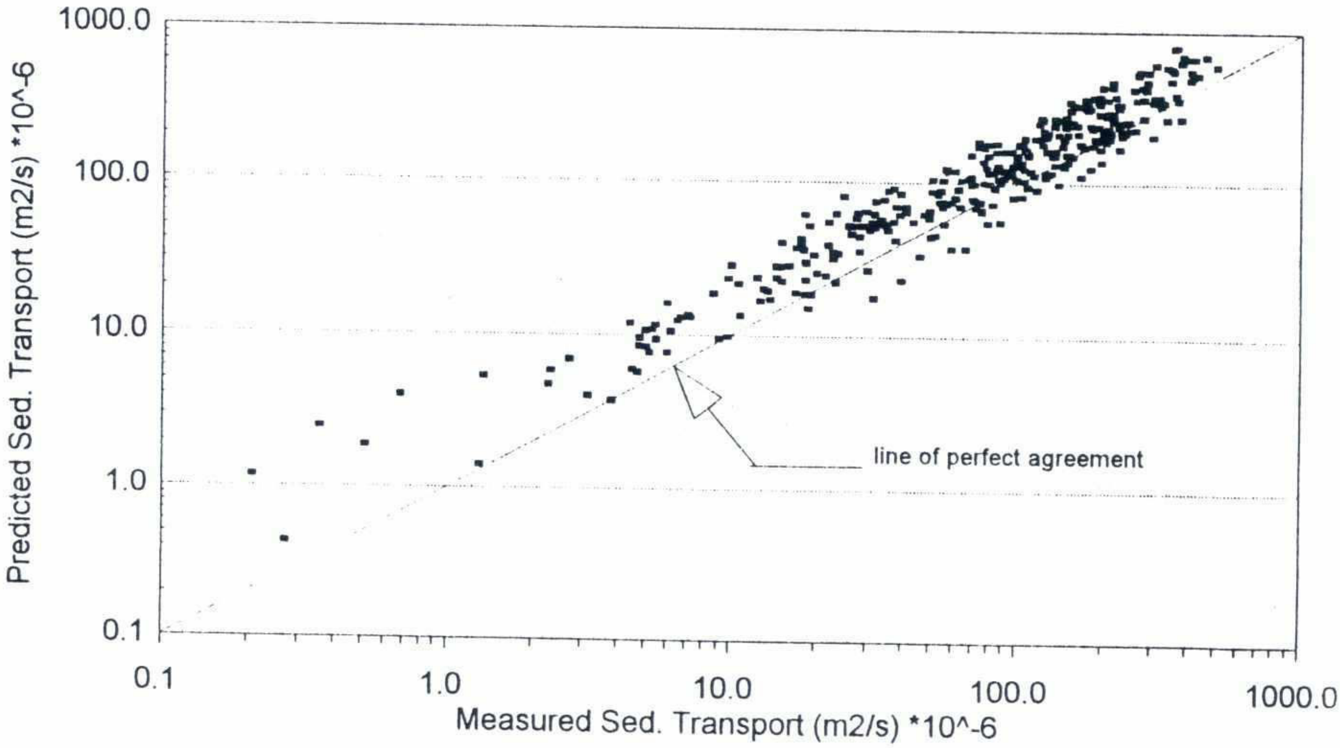


Figure 5.10: Verification of Suggested Equation against 1993 data

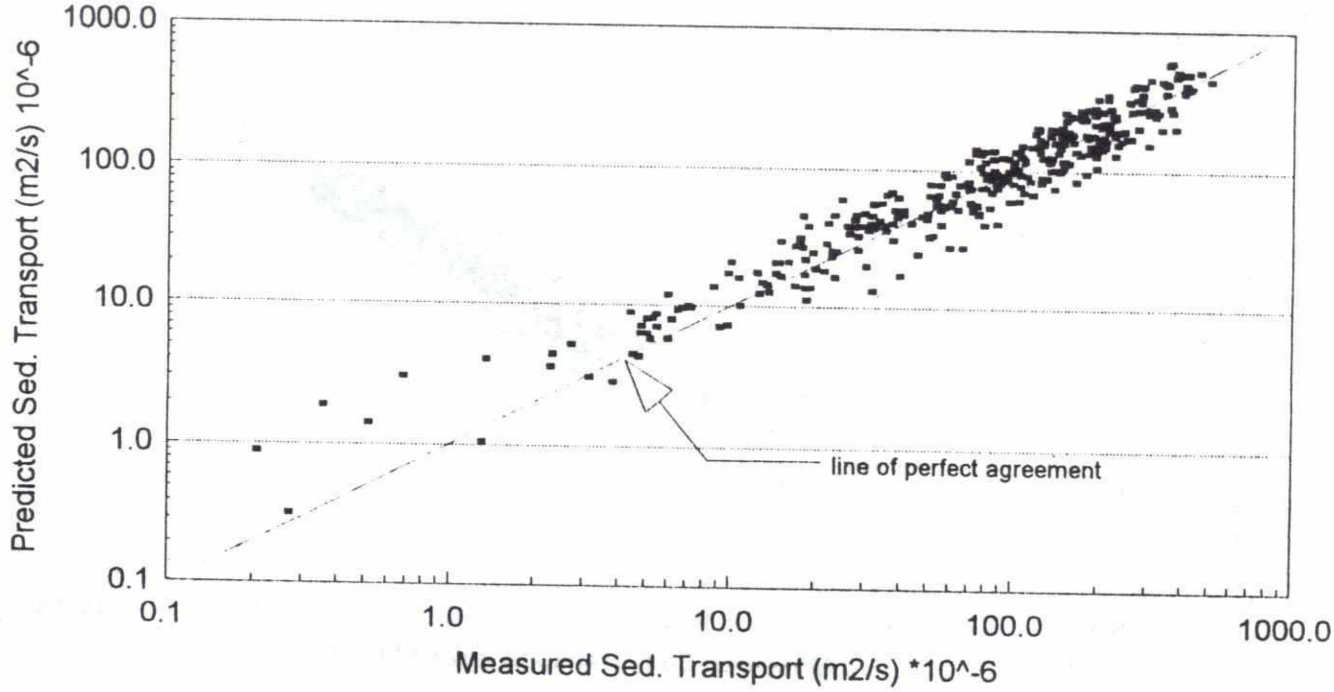


Figure 5.11: Verification of Suggested Equation (corrected) against 1993 data

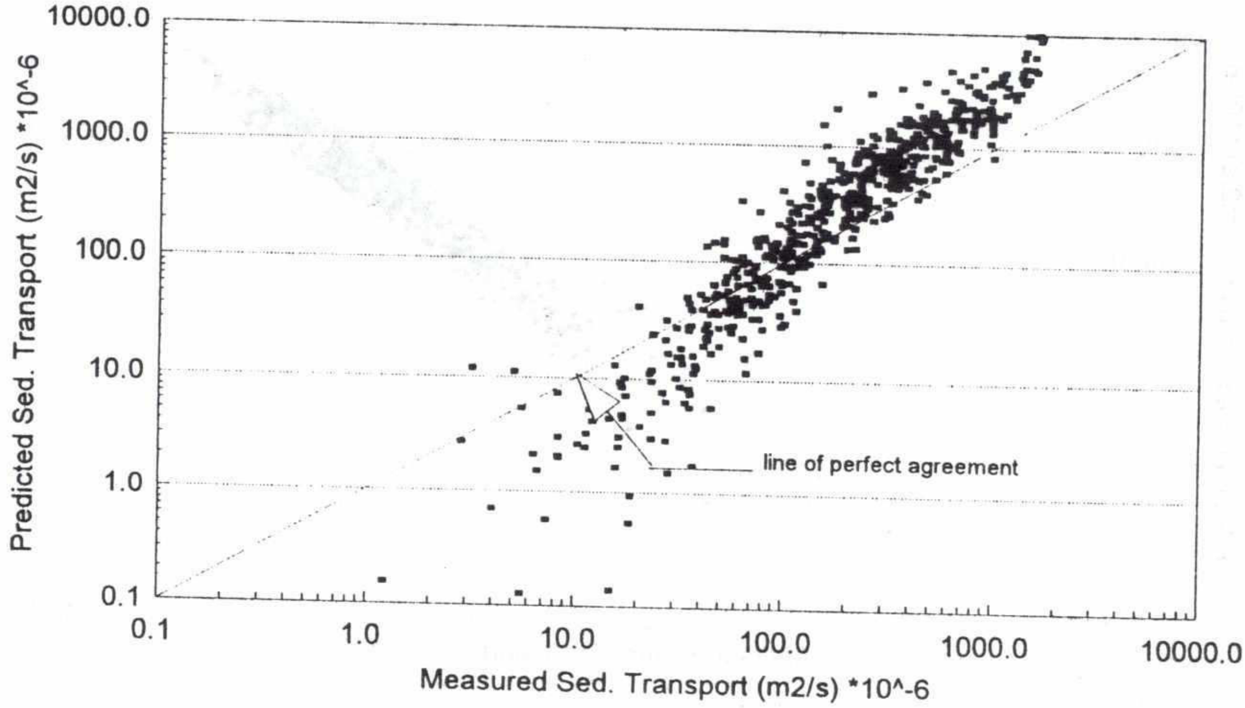


Figure 5.12: Predicted and measured sediment transport, Ackers & White formula

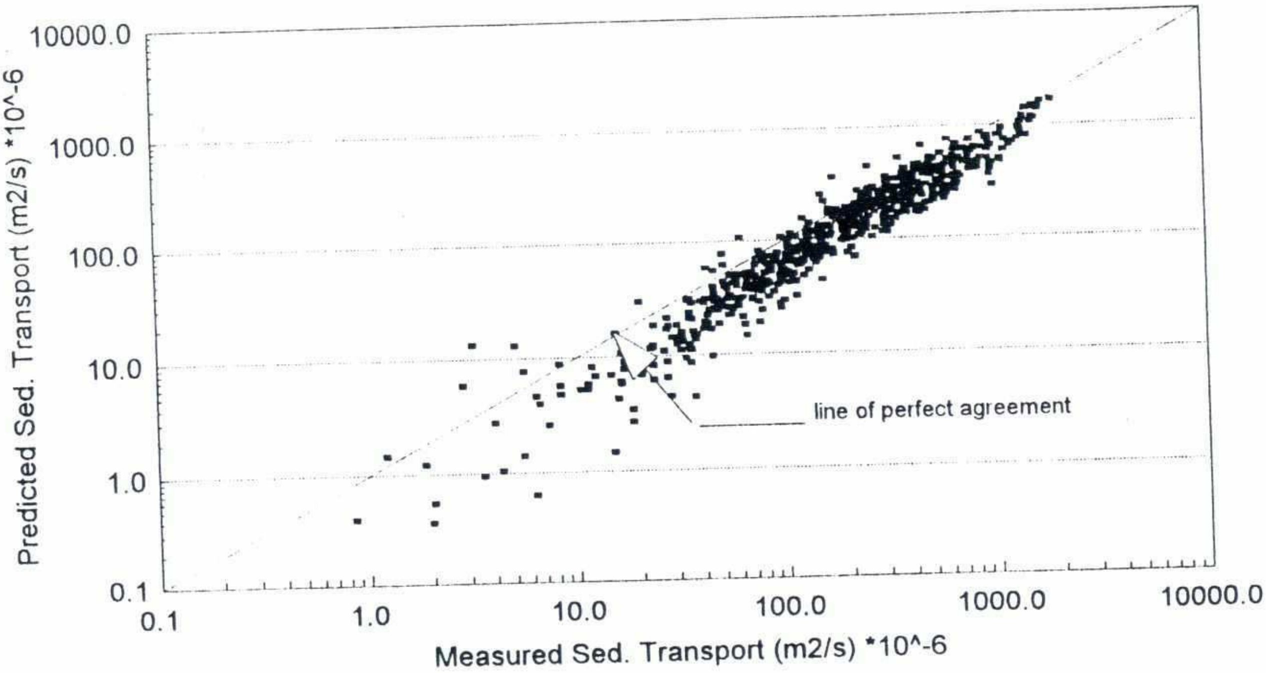


Figure 5.13: Predicted and measured sediment transport, Bagnold formula

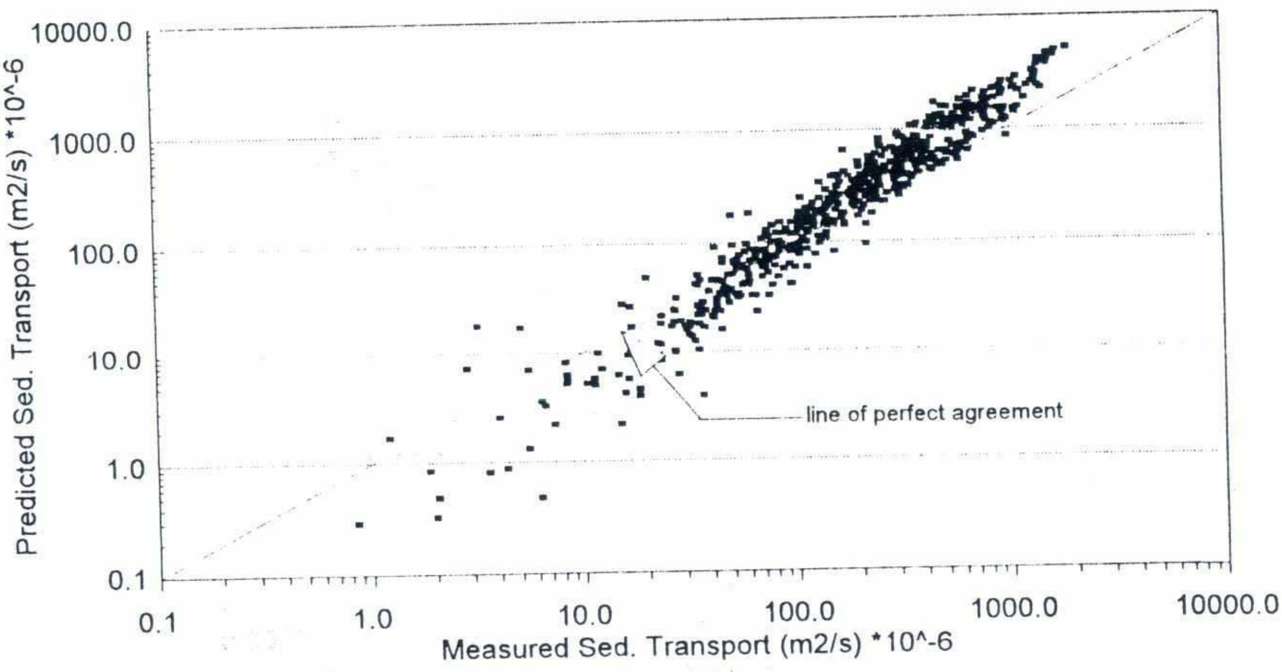


Figure 5.14: Predicted and measured sediment transport, Engelund-Hansen formula

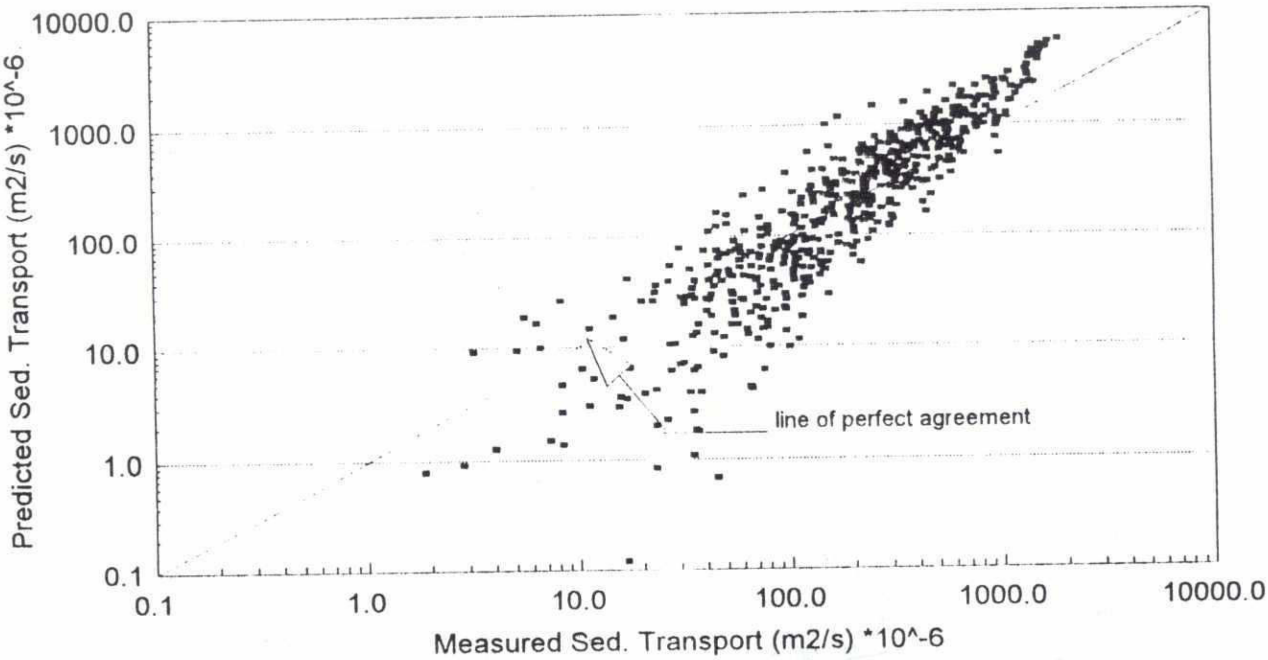


Figure 5.15: Predicted and measured sediment transport, van Rijn formula

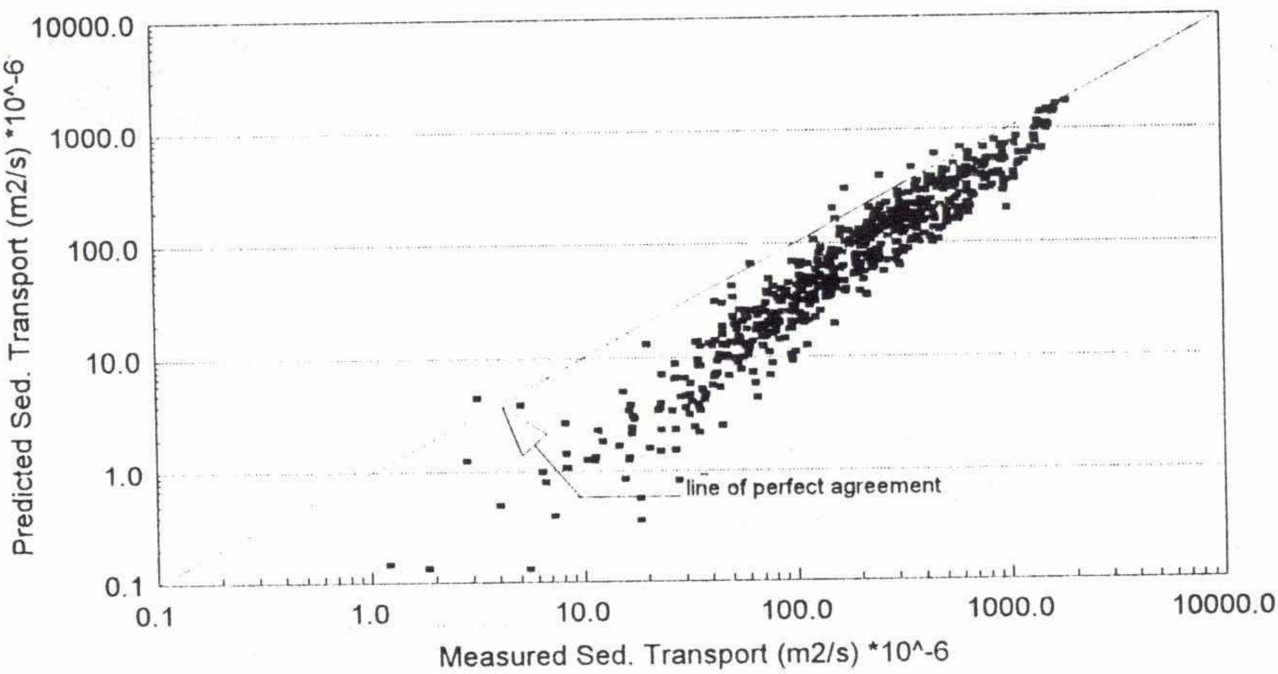


Figure 5.16: Predicted and measured sediment transport, Yang formula

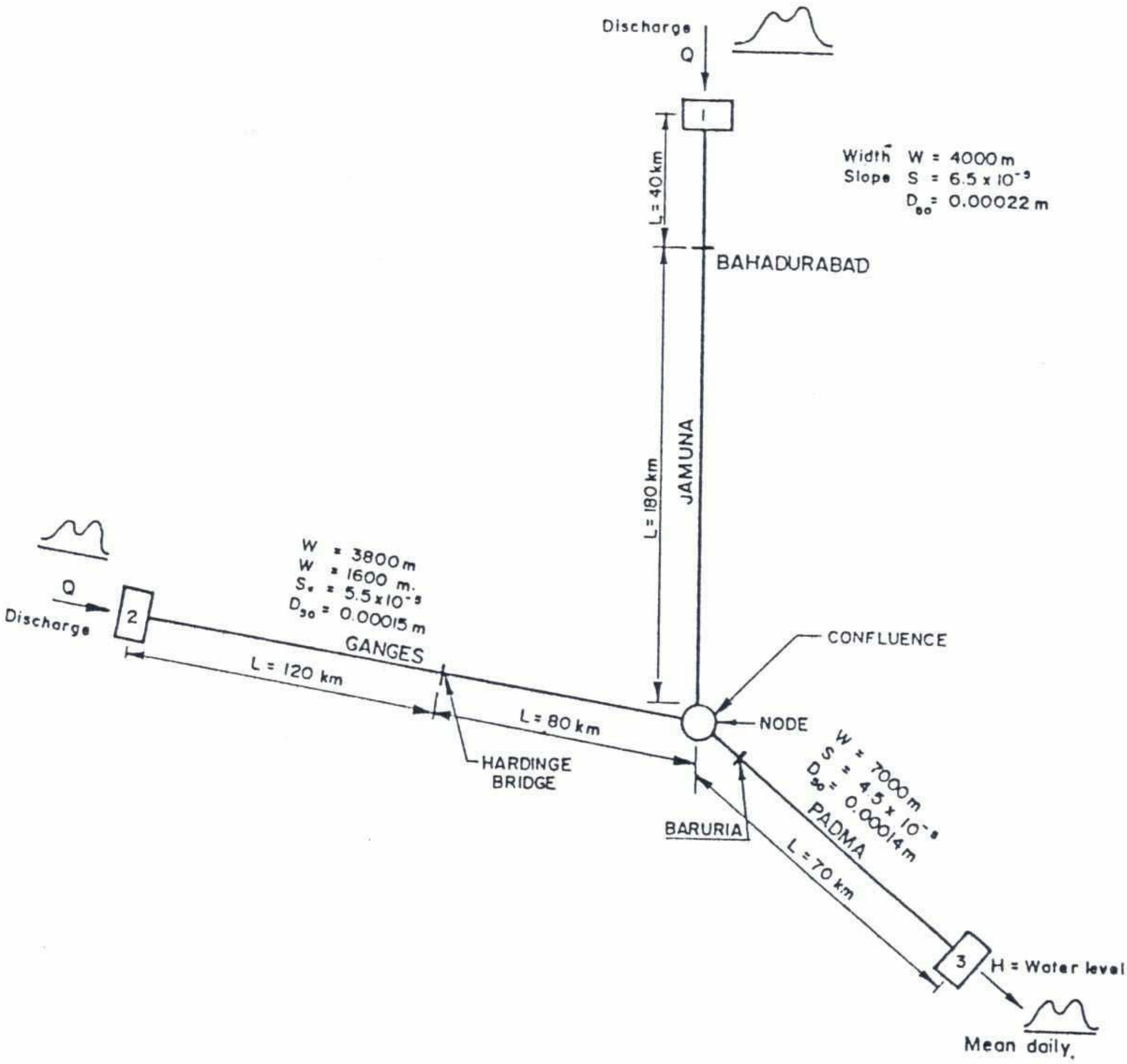


Figure 6.1: Network schematization of the GJP model

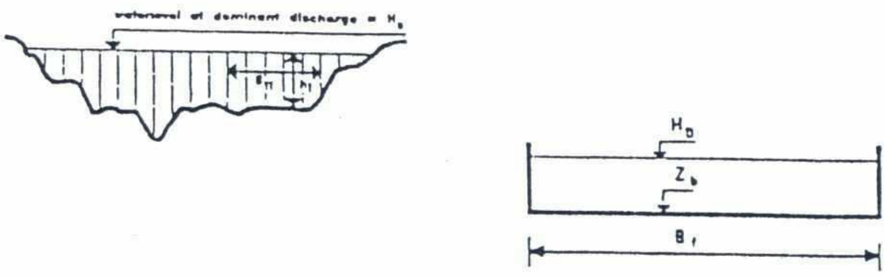


Figure 6.2: Cross-section schematization of the GJP model

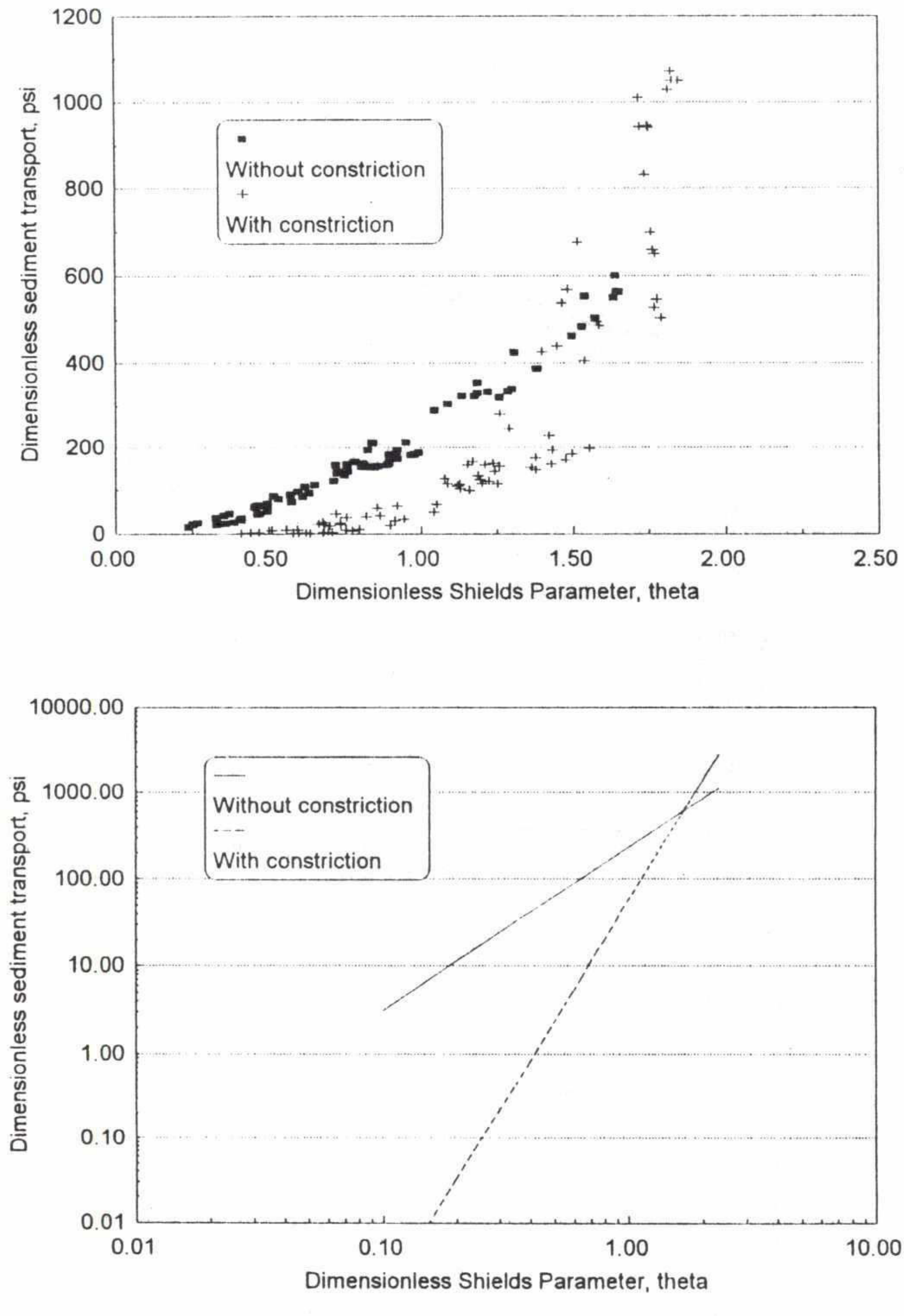


Figure 6.3: Comparison of dimensionless sediment rating curves, Ganges River

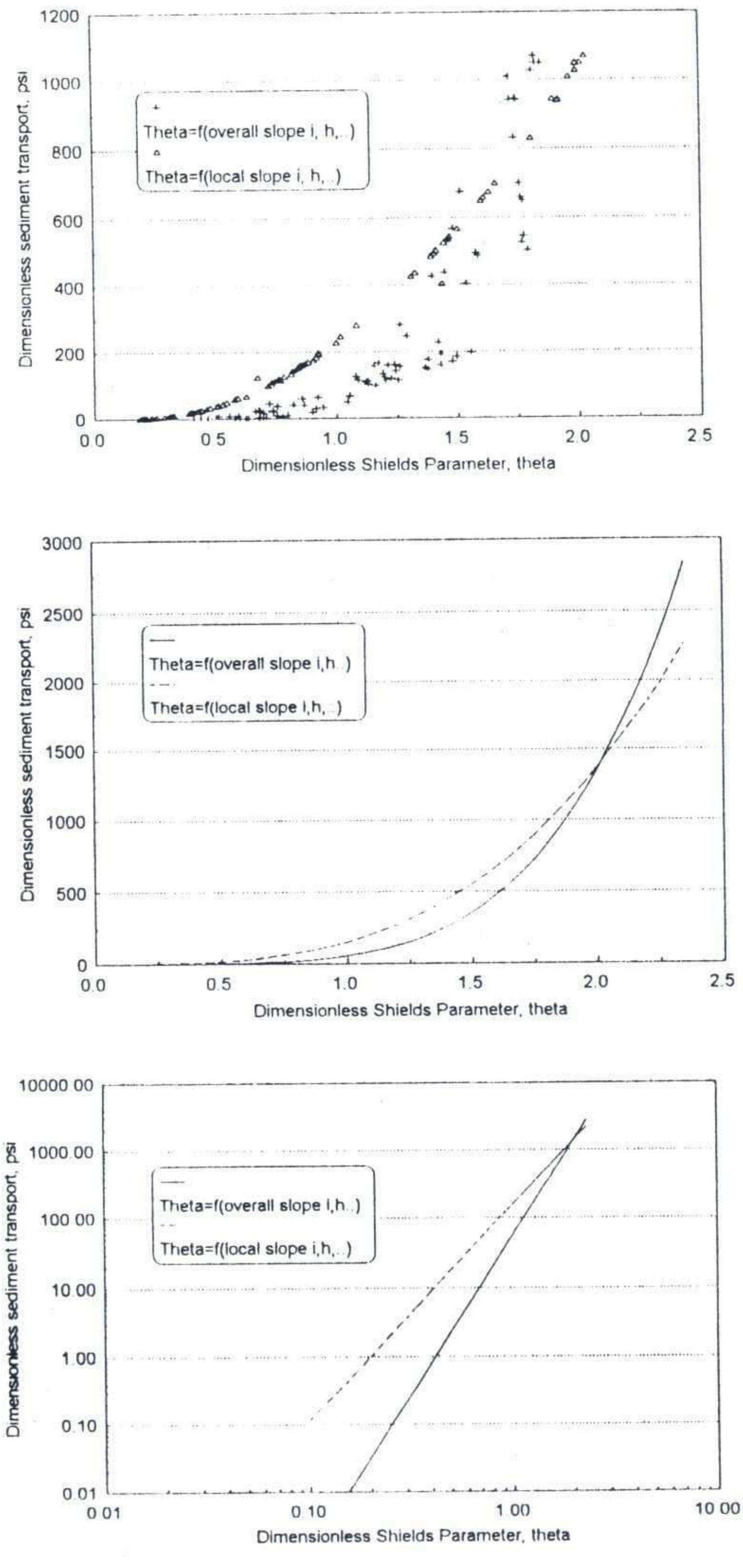


Figure 6.4: Comparison of dimensionless sediment rating curves at the constriction

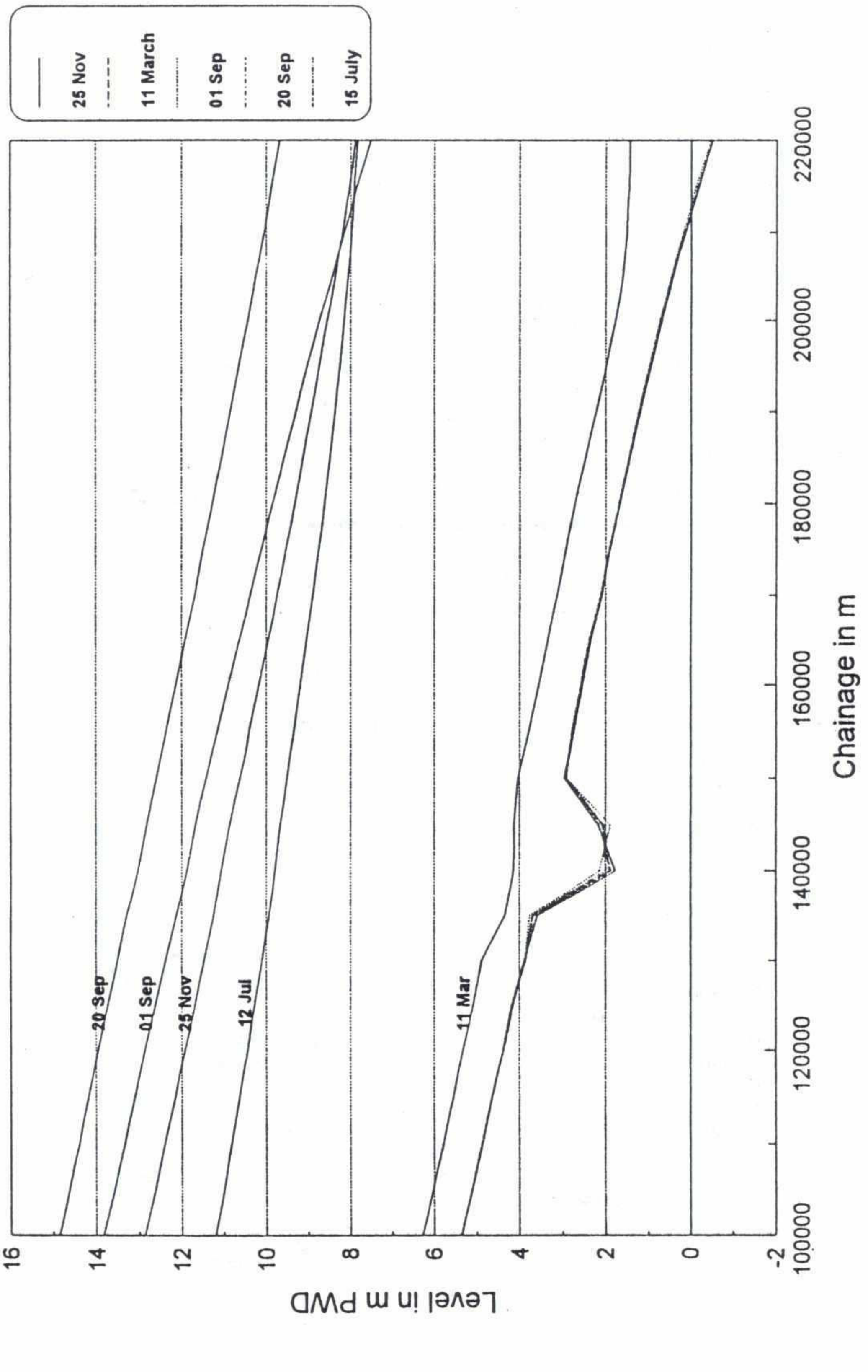
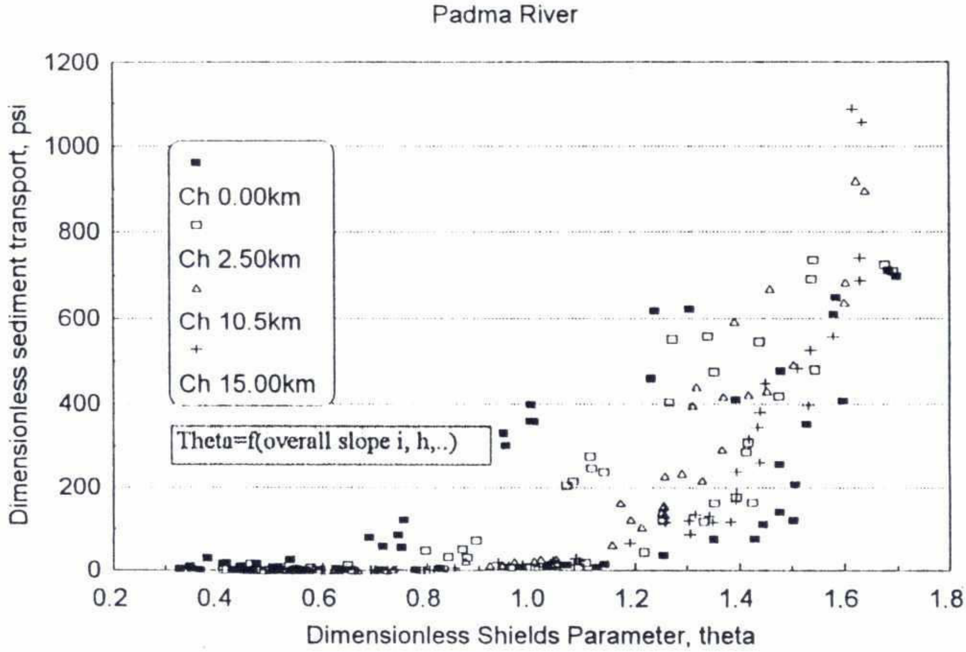


Figure 6.5: Calculated variation of longitudinal bed and water level profiles, Ganges River



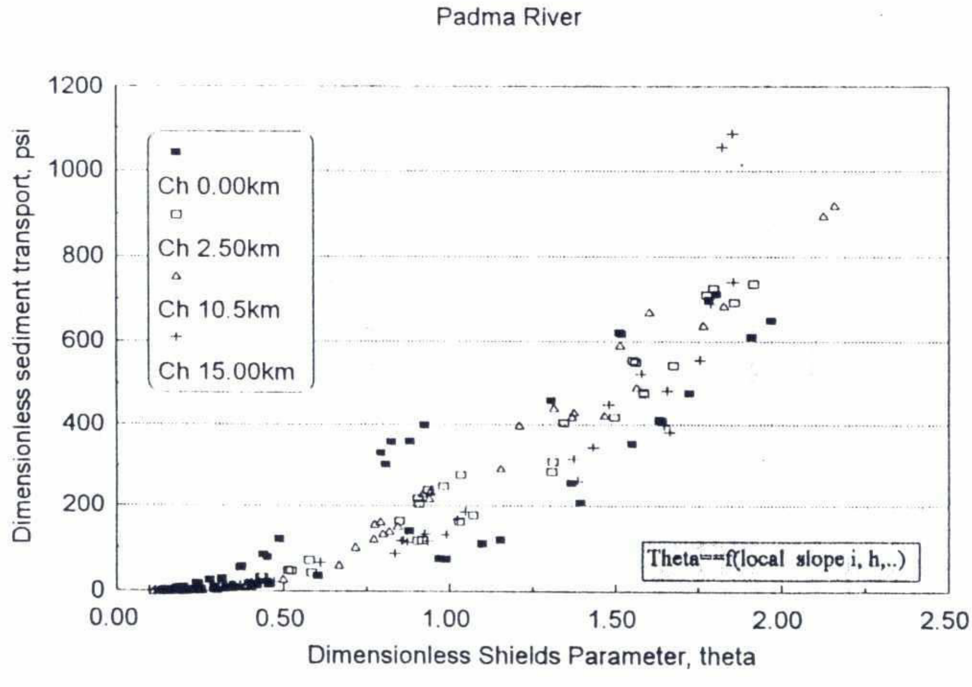
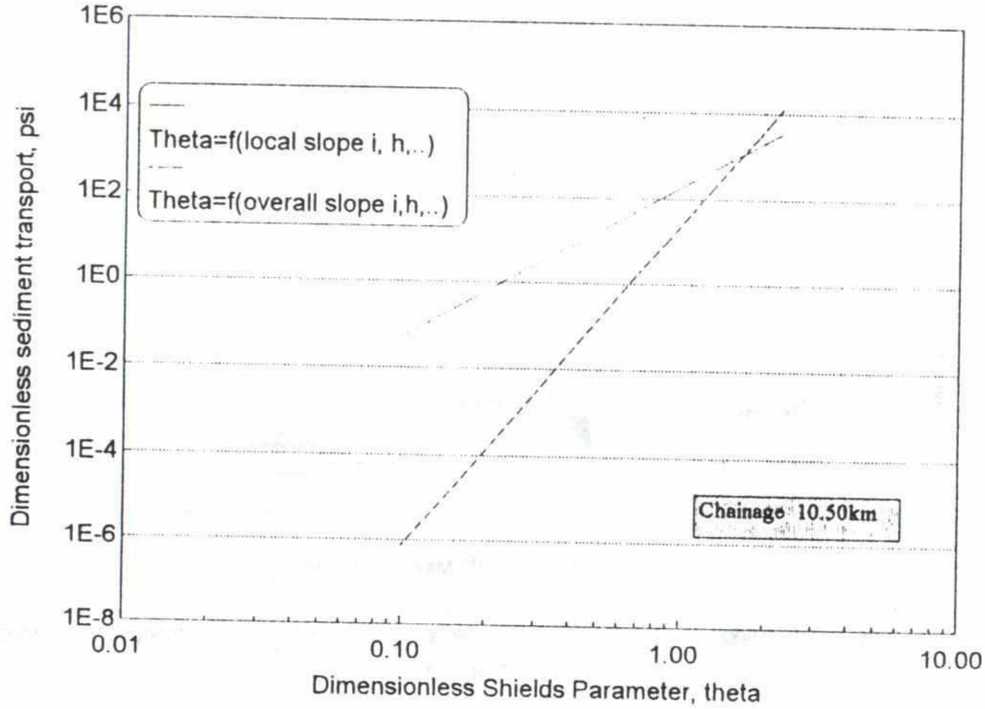


Figure 6.6: Effect of over-all and local slope on the sediment transport, Padma River



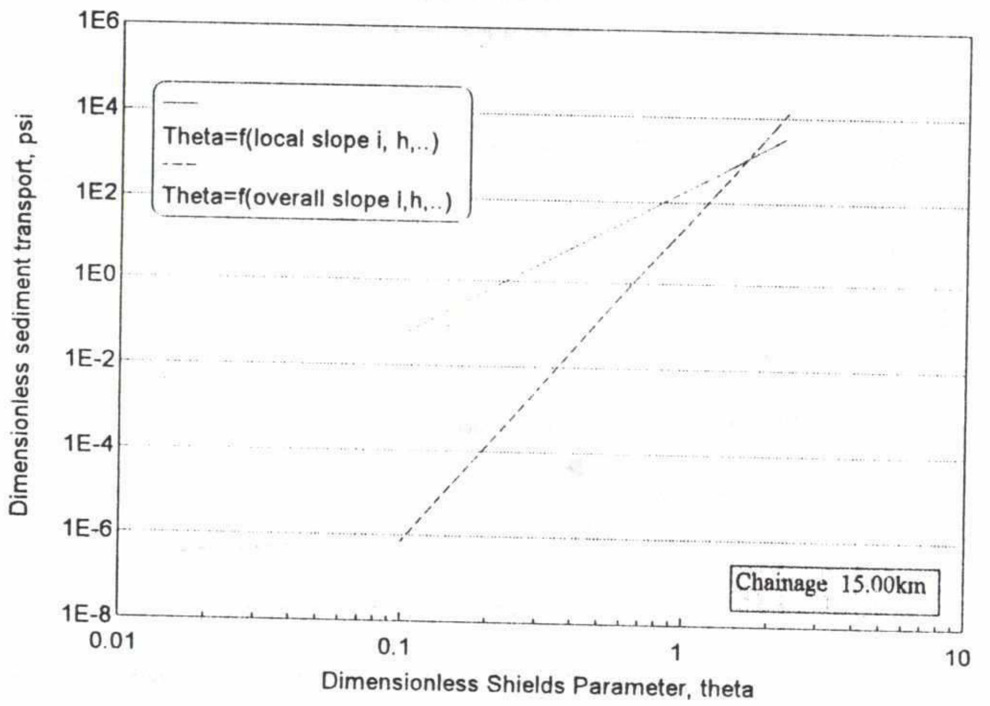


Figure 6.7: Dimensionless sediment rating curves, Padma River

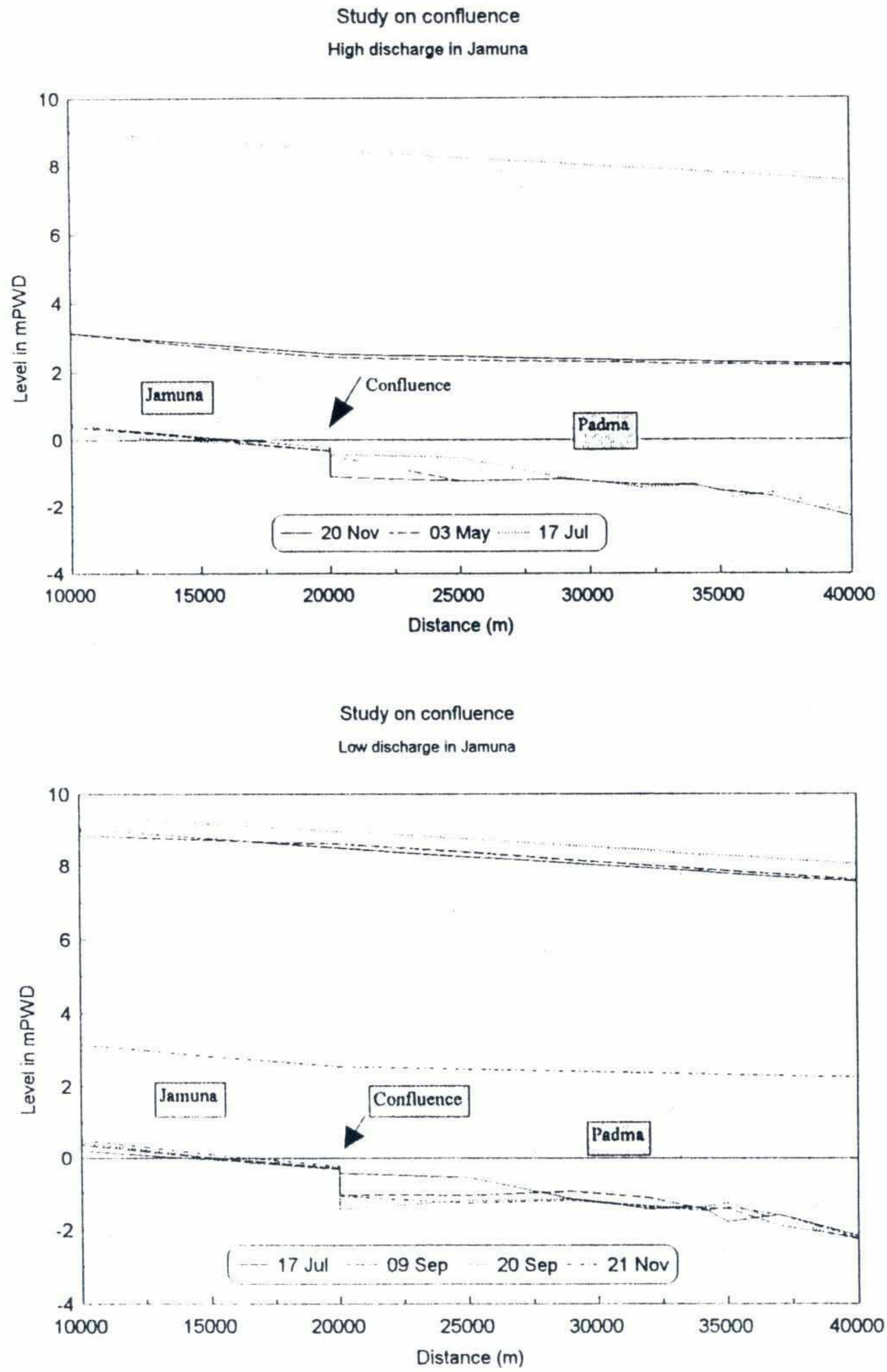


Figure 6.8: Dynamic equilibrium around a confluence

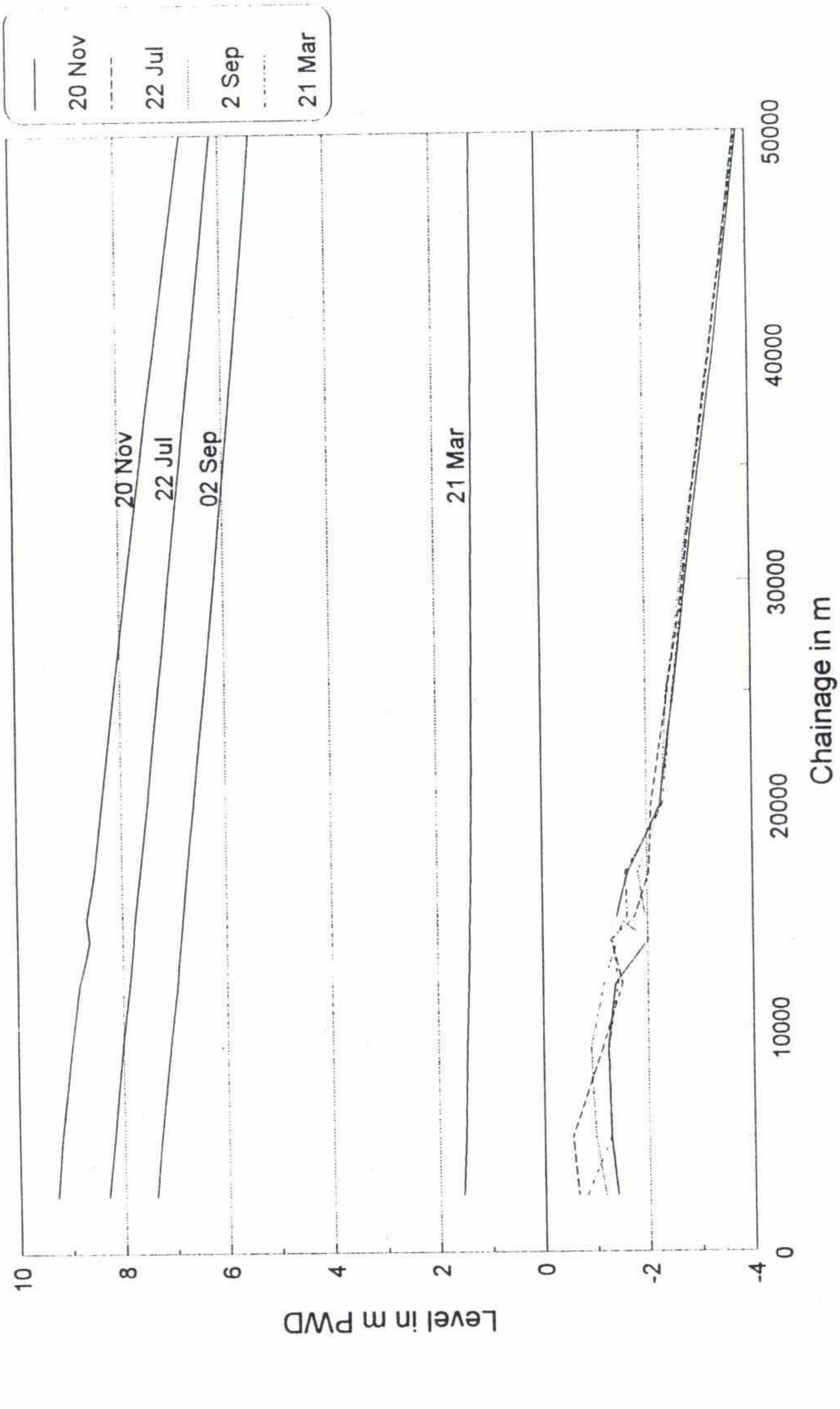


Figure 6.9: Calculated variation of longitudinal bed and water level profiles, Padma River

Special Report 13, Annexure 1

Derivation of exponent 'n' from different prediction formulae

October 1996

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1. Introduction

This annexure mainly deals with the derivation of the velocity exponent 'n' for the selected prediction formula. These formulae were selected on the basis of criteria such as their suitability for application in alluvial rivers, and the sediment transport being described as a function of the bed shear stress under the influence of accelerating and decelerating flow conditions.

2. Basis for the derivation

The schematised sediment transport equation can be expressed as:

$$s = m\, u^{n} \qquad \text{(A.1)}$$

Differentiation of this expression with respect to 'u' gives

$$\frac{ds}{du} = \frac{n}{u}\,(m\, u^{n}) = \frac{n}{u}\, s \qquad \text{(A.2)}$$

Therefore, the expression of exponent 'n' can be written as:

$$n = \frac{ds}{du} \cdot \frac{u}{s} \qquad \text{(A.3)}$$

After derivation of 'n' from each prediction formula, the next step is to relate 'n' to the Shields parameter (θ). This parameter represents most of the hydraulic parameters (e.g slope, velocity, grain size etc.) that influence the sediment transport.

3. Derivation from selected total load formulae

Six total load prediction formulae have been selected for examination. These are: Ackers and White, Bagnold, Colby, Engelund-Hansen, Van Rijn, and Yang. In the following sections, details are given on the derivation of 'n' in each case.

3.1 Ackers and White (1973)

The total load formula for Ackers and White reads as:

$$s_t = k\, u\, d_{35}\, \left(\frac{u}{u_*}\right)^{n'} \left(\frac{Y - Y_{cr}}{Y_{cr}}\right)^{m'} \qquad \text{(A.4)}$$

Substituting the bed-shear velocity 'u_*' equal to $g^{1/2}$ u/C, and considering ψ equal to Y/Y_{cr} give

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$$s_t = k\ d_{35}\ (\frac{C}{\sqrt{g}})^{n'} (\psi-1)^{m'}\ u \tag{A.5}$$

Differentiation of this equation with respect to u gives

$$\frac{ds_t}{du} = k\ d_{35}\ (\frac{C}{\sqrt{g}})^{n'} [\ m'\ (\psi-1)^{m'-1}\ \frac{d\psi}{du}\ u + (\psi-1)^{m'}] \tag{A.6}$$

By inserting equation (A.3) into (A.6), the following expressions can be obtained:

$$k\ d_{35}(\frac{C}{\sqrt{g}})^{n'} [\ m'\ (\psi-1)^{m'-1}\ \frac{d\psi}{du}\ u + (\psi-1)^{m'}] = \frac{n}{u}\ s_t \tag{A.7}$$

$$\frac{d\psi}{du} = \frac{d}{du}(\frac{Y}{Y_{cr}}) = \frac{1}{Y_{cr}}\ \frac{dY}{du} \tag{A.8}$$

The particle mobility parameter, Y, can be expressed as:

$$Y = (\frac{u_*^{n'}}{\sqrt{(s-1)\ g\ d_{35}}})\ (\frac{u}{5.66\ \log\ (\frac{10h}{d_{35}})})^{1-n'} \tag{A.9}$$

Substituting the bed shear velocity 'u*' equal to $g^{1/2}u/C$ in equation (A.9) and then simplifying yields

$$Y = \frac{1}{\sqrt{(s-1)\ g\ d_{35}}}(\frac{\sqrt{g}}{C})^{n'}\ \frac{u}{[5.66\ \log\ (\frac{10h}{d_{35}})]^{1-n'}} \tag{A.10}$$

Differentiating equation (A.10) with respect to 'u' and substituting into equation (A.8) yields

$$\frac{d\psi}{du} = \frac{1}{Y_{cr}} \cdot \frac{1}{\sqrt{(s-1)\ g\ d_{35}}}\ (\frac{\sqrt{g}}{C})^{n'}\ \frac{1}{[5.66\ \log\ (\frac{10h}{d_{35}})]^{1-n'}} \tag{A.11}$$

Combining equations (A.5), (A.7) and (A.11) yields the exponent 'n' as

$$n = 1 + \frac{m'}{1 - \frac{Y_{cr}}{Y}} \tag{A.12}$$

in which:

$$Y = \left(\frac{\sqrt{g}}{C}\right)^{n'-1} \sqrt{\frac{\theta d_{50}}{d_{35}}} \left(\frac{1}{5.66\log[\frac{10h}{d_{35}}]}\right)^{1-n'} \tag{A.13}$$

3.2 Bagnold (1966)

The total load formula of Bagnold reads as follows:

$$s_t = \frac{\tau_b \, u}{(\rho_s - \rho)\, g \cos\beta} \left[\frac{e_s(1-e_b)}{\frac{w_s}{u} - \tan\beta} + \frac{e_b}{(\tan\phi - \tan\beta)}\right] \tag{A.14}$$

In equation (A.14), the bed slope, $\tan\beta$ (= I_b) is much less than the value of the dynamic friction coefficient, $\tan\phi$ (= 0.6), as well as much less than the ratio of the fall velocity to the bed-shear velocity (w_s/u_*). Therefore neglecting $\tan\beta$ and substituting the bed shear stress τ_b equal to $\rho g h I$ in equation (A.14) yields

$$s_t = \frac{u^3}{C^2\Delta\cos\beta} \left[\frac{e_s\,(1 - e_b)}{w_s} + \frac{e_b}{\tan\phi}\right] \tag{A.15}$$

Now considering

$$\psi = \frac{e_s\,(1 - e_b)\, u}{w_s} + \frac{e_b}{\tan\phi} \tag{A.16}$$

Then equation (A.15) yields

$$s_t = \frac{u^3}{C^2\Delta\cos\beta}\, \psi \tag{A.17}$$

Differentiation of (A.17) with respect to 'u' gives

$$\frac{ds_t}{du} = \frac{1}{C^2\Delta\cos\beta} \left[3u^2\psi + u^3\, \frac{e_s(1-e_b)}{w_s}\right] \tag{A.18}$$

Again applying equation (A.3) in equation (A.18) yields

$$\frac{1}{C^2\Delta\cos\beta} \left[3u^2\psi + u^3\, \frac{e_s(1-e_b)}{w_s}\right] = \frac{n}{u}\, s_t \tag{A.19}$$

Combining equations (A.17) and (A.19), and with 'u' equal to $C(\Delta d_{50}\,\theta)^{1/2}$, it is possible to obtain the exponent 'n' as

$$n = 3 + \cfrac{1}{1 + \dfrac{e_b}{e_s(1 - e_b)\tan\phi} \cdot \dfrac{w_s}{C\sqrt{\Delta d_{50}\theta}}} \qquad \text{(A.20)}$$

3.3 Colby (1957)

Colby studied the variation of the sediment load of several streams in USA. He has found that the load, s_s (kN/day/m), increases with the mean velocity of the flow, u (m/s), and that the relationship between these two parameters can be expressed as follows:

$$s_s = 353.6\ u^{3.1} \qquad \text{(A.21)}$$

The exponent of the flow velocity, 'n', can be read directly as 3.1.

3.4 Engelund-Hansen (1967)

The total load formula of Engelund-Hansen reads as:

$$s_t = \frac{0.05\ u^5}{(s - 1)^2\ g^{0.5}\ d_{50}\ C^3} \qquad \text{(A.22)}$$

The exponent of the flow velocity, 'n', can be read directly as 5.

3.5 Van Rijn (1984)

Van Rijn expresses the total load transport (s_t) as

$$s_t = F\ u\ h\ c_a + 0.1\ (s-1)^{0.5} g^{0.5} d_{50}^{\ 1.5} \frac{T^{1.5}}{D_*^{\ 0.3}} \qquad \text{(A.23)}$$

The reference concentration c_a can be expressed as:

$$c_a = 0.015\ \frac{d_{50}}{a}\ \frac{T^{1.5}}{D_*^{0.3}} \qquad \text{(A.24)}$$

Substituting the value of c_a in equation (A.23) and then simplifying gives

$$s_t = A\ T^{1.5}\ u^3 \qquad \text{(A.25)}$$

whereby A is considered constant for the sake of simplification of the derivation. A is given by the following expression:

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$$A = \frac{1}{C^2 I} [0.015\ F \frac{d_{50}}{a} \frac{1}{D_*^{0.3}} + 0.10 \frac{d_{50}}{D_*^{0.3}} \frac{1}{Ch} \sqrt{\frac{g}{\theta}}\ ] \tag{A.26}$$

The transport stage parameter T can be expressed as:

$$T = \frac{\tau_b^{/} - \tau_{b,cr}}{\tau_{b,cr}} \tag{A.27}$$

Substituting T in equation (A.25) and considering ψ equal to $\tau_b'/\tau_{b,cr}$ yields

$$q_t = A\ (\psi-1)^{1.5}\ u^3 \tag{A.28}$$

By differentiation of equation (A.28) with respect to 'u', it is possible to obtain the following expression:

$$\frac{ds_t}{du} = A\ [\frac{3\rho g}{C^{/2}\tau_{b,cr}}\ (\psi-1)^{0.5}\ u^4 + 3\ (\psi-1)^{1.5}\ u^2] \tag{A.29}$$

Now, applying equation (A.3) in equation (A.29) gives

$$A\ [\frac{3\rho g}{C^{/2}\tau_{b,cr}}\ (\psi-1)^{0.5}\ u^4 + 3\ (\psi-1)^{1.5}\ u^2] = \frac{n}{u}\ s_t \tag{A.30}$$

By combining equations (A.28) and (A.30), it is possible to express exponent 'n' as

$$n = 3 + \frac{3}{1-\frac{\theta_{cr}}{\theta}(\frac{C^{/}}{C})^2} \tag{A.31}$$

3.6 Van Rijn (1984), simplified form

The simplified form of the Van Rijn formula for total load transport can be expressed as

$$s_t = (\frac{u-u_{cr}}{\sqrt{(s-1)g\ d_{50}}})^{2.4}\ [0.012\ uh\ \frac{d_{50}}{h}\ (\frac{1}{D_*})^{0.6} + 0.005\ uh\ (\frac{d_{50}}{h})^{1.2}] \tag{A.32}$$

The above expression can be further simplified in the following form

$$s_t = A\ (1-\frac{u_{cr}}{u})^{2.4}\ u^3 \tag{A.33}$$

In the above formula, A is considered constant for the sake of simplification of the derivation. A is given by the following expression

$$A = \frac{1}{C^2 I} \left(C \sqrt{\frac{\theta}{g}}\right)^{2.4} \left[0.012 \frac{d_{50}}{h} \left(\frac{1}{D_*}\right)^{0.6} + 0.005 \left(\frac{d_{50}}{h}\right)^{1.2}\right] \qquad (A.34)$$

Now, considering ψ equal to $(1-u_{cr}/u)$, equation (A.33) gives

$$s_t = A\ \psi^{2.4}\ u^3 \qquad (A.35)$$

By differentiation of equation (A.35) with respect to 'u', it is possible to obtain

$$\frac{ds_t}{du} = A\ (2.4\ u_{cr}\ \psi^{1.4}\ u + 3\ \psi^{2.4}\ u^2] \qquad (A.36)$$

Now, applying equation (A.3) in equation (A.36) yields

$$A\ (2.4\ u_{cr}\ \psi^{1.4}\ u + 3\ \psi^{2.4}\ u^2] = \frac{n}{u}\ s_t \qquad (A.37)$$

Combining equations (A.35) and (A.37), and 'u' equal to $(\Delta d_{50}\theta)^{1/2}$, the exponent 'n' is derived as

$$n = 3 + \frac{2.4}{C\ \frac{\sqrt{\Delta d_{50}\theta}}{u_{cr}} - 1} \qquad (A.38)$$

In which the critical velocity, u_{cr} according to Shields can be obtained by the following expressions

$$u_{cr} = 8.50\ (d_{50})^{0.6}\ \log\left(\frac{12h}{3d_{90}}\right) \quad for \quad 0.0005 \le d_{50} \le 0.002 \qquad (A.39)$$

$$u_{cr} = 0.19\ (d_{50})^{0.1}\ \log\left(\frac{12h}{3d_{90}}\right) \quad for \quad 0.0001 \le d_{50} \le 0.0005 \qquad (A.40)$$

3.7 Yang (1973)

The total sediment concentration, c_t reads as

$$\log\ (c_t) = \alpha_1 + \alpha_2\ \log\ \left(\frac{uI - u_{cr}I}{w_s}\right) \qquad (A.41)$$

This equation can be rewritten as:

$$c_t = 10^{\alpha_1} . (\frac{uI - u_{cr}I}{w_s})^{\alpha_2} \tag{A.42}$$

The total load transport s_t can be expressed as:

$$s_t = 10^{-3} \ c_t \ u \ h \tag{A.43}$$

Substituting the value of c_t in equation (A.43) and applying 'I' equal to u^2/C^2h gives

$$s_t = 10^{\alpha_1 -3} \ h \ [\frac{1}{C^2.h.w_s}]^{\alpha_2} \ (u - u_{cr})^{\alpha_2} \ u^{2\alpha_2 +1} \tag{A.44}$$

In practical cases, the critical depth-averaged velocity, u_{cr}, is much less than the depth-averaged velocity 'u'. Therefore, neglecting the term u_{cr}, the above expression can be rewritten as

$$s_t = 10^{\alpha_1 -3} \ h \ [\frac{1}{C^2.h.w_s}]^{\alpha_2} \ u^{3\alpha_2 +1} \tag{A.45}$$

Differentiating equation (A.45) with respect to 'u' and applying equation (A.3) yields

$$(3\alpha_2 + 1) \ 10^{\alpha_1 -3} \ h \ [\frac{1}{C^2.h.w_s}]^{\alpha_2} \ u^{3\alpha_2} = \frac{n}{u} \ s_t \tag{A.46}$$

Combining equations (A.45) and (A.46) gives the exponent 'n' as

$$n = 3\alpha_2 + 1 \tag{A.47}$$

The parameter α_2 can be expressed as:

$$\alpha_2 = 1.799 - 0.409 \log (\frac{w_s d_{50}}{\nu}) - 0.314 \log (\frac{u_*}{w_s}) \tag{A.48}$$

Substituting the parameter α_2 in equation (A.48), and with the bed shear velocity u_* equal to $(g\Delta d_{50}\theta)^{1/2}$, equation (A.47) yields

$$n = 6.397 - 1.227 \log (\frac{w_s d_{50}}{\nu}) - 0.942 \log (\frac{\sqrt{g\Delta d_{50}\theta}}{w_s}) \tag{A.49}$$

4. Derivation from selected bed load formulae

Three bed load prediction formulae have been selected for derivation of the exponent 'n'. These are Meyer-Peter and Mueller, two forms of the Van Rijn formula, and the Parker & Klingeman formula. The details of the derivations for each formula are outlined in the following sections.

4.1 Meyer-Peter and Mueller (1948)

The bed load transport formula in its dimensionless form can be expressed as:

$$\phi_b = 8\ (\mu\ \theta\ -\ 0.047)^{1.5} \tag{A.50}$$

Substituting ϕ_b equal to $s_b/(\Delta g d_{50}^{3})^{1/2}$, and considering ψ equal to $\mu\theta$, the above expression yields

$$s_b = 8\ \sqrt{\Delta\ g\ d_{50}^{3}}\ (\psi\ -\ 0.047)^{1.5} \tag{A.51}$$

Differentiation with respect to 'u' gives

$$\frac{ds_b}{du} = 24\ \frac{\mu\ \sqrt{\Delta\ g\ {d_{50}}^{3}}}{C^2 \Delta d_{50}}\ (\psi - 0.047)^{0.5}\ u \tag{A.52}$$

Applying equation (A.3) in equation (A.52) yields

$$24\ \frac{\mu\ \sqrt{\Delta\ g\ {d_{50}}^{3}}}{C^2 \Delta d_{50}}\ (\psi - 0.047)^{0.5}\ u = \frac{n}{u}\ s_b \tag{A.53}$$

Combining equations (A.51) and (A.53) yields the exponent 'n' as

$$n = \frac{3}{1 - \dfrac{0.047}{\mu\theta}} \tag{A.54}$$

Here, it is worthwhile to mention that the above expression is similar to the expression derived by de Vries (1987), except for the parameter μ.

4.2 Van Rijn (1984), original form

The bed load transport formula for Van Rijn reads as:

$$s_b = 0.10\ (s-1)^{0.5}\ g^{0.5}\ d_{50}^{1.5}\ D_*^{-0.3}\ T^{1.5} \qquad For\ \ T \geq 3 \tag{A.55}$$

This equation can be rewritten as:

$$s_b = 0.10\ d_{50}\ g^{0.5} \sqrt{\Delta d_{50}}\ \frac{T^{1.5}}{D_*^{\ 0.3}} \tag{A.56}$$

Substituting $(\Delta d_{50})^{1/2}$ equal to $u/C\theta^{1/2}$ and T equal to $(\tau_b'/\tau_{b,cr}-1)$, the above expression yields

$$s_b = A\ (\frac{\tau_b'}{\tau_{b,cr}} - 1)^{1.5}\ u \tag{A.57}$$

where A is considered constant for the sake of simplification of derivation. A is given by the following expression:

$$A = 0.10 \sqrt{\frac{g}{\theta}}\ \frac{d_{50}}{C\ D_*^{\ 0.3}} \tag{A.58}$$

Considering ψ equal to $\tau_b'/\tau_{b,cr}$ and differentiating equation (A.57) with respect to 'u' gives

$$\frac{ds_b}{du} = A\ [\frac{3\rho g}{\tau_{b,cr}\ C'^2}\ (\psi-1)^{0.5}\ u^2 + (\psi-1)^{1.5}] \tag{A.59}$$

Applying equation (A.3) in equation (A.59) yields

$$A\ [\frac{3\rho g}{\tau_{b,cr}\ C'^2}\ (\psi-1)^{0.5}\ u^2 + (\psi-1)^{1.5}] = \frac{n}{u}\ s_b \tag{A.60}$$

Combining equations (A.57) and (A.60), it is possible to obtain the exponent 'n' as

$$n = 1 + \frac{3}{1 - \frac{\theta_{cr}}{\theta}\ (\frac{C'}{C})^2} \tag{A.61}$$

4.3 Van Rijn (1984), simplified form

The bed load transport formula of Van Rijn equation in its simplified form can be expressed as:

$$s_b = 0.005\ u\ h\ (\frac{u - u_{cr}}{\sqrt{(s-1)\ g\ d_{50}}})^{2.4}\ (\frac{d_{50}}{h})^{1.2} \tag{A.62}$$

Substituting 'h' equal to u^2/C^2I and considering ψ equal to $(1-u_{cr}/u)$, the above expression gives

$$s_b = A\ \psi^{2.4}\ u^{0.6} \tag{A.63}$$

where A is assumed constant for the sake of simplification of the derivation. A is given by the following expression:

$$A = 0.005\ d_{50}^{1.2}\ \frac{(C^2 I)^{0.2}}{\left(\sqrt{\frac{g}{C^2\theta}}\right)^{2.4}} \tag{A.64}$$

Now, differentiation of equation (A.63) with respect to 'u' gives

$$\frac{ds_b}{du} = A\ [0.6\ \frac{\psi^{2.4}}{u^{0.4}} + 2.4\ \psi^{1.4}\ \frac{u_{cr}}{u^{1.4}}] \tag{A.65}$$

Applying equation (A.3) in equation (A.65) yields

$$A\ [0.6\ \frac{\psi^{2.4}}{u^{0.4}} + 2.4\ \psi^{1.4}\ \frac{u_{cr}}{u^{1.4}}] = \frac{n}{u}\ s_b \tag{A.66}$$

Now, combining equations (A.63) and (A.66), and with 'u' equal to $C(\Delta d_{50}\theta)^{1/2}$, it is possible to obtain the exponent 'n' as

$$n = 0.6 + \frac{2.4}{C\ \frac{\sqrt{\Delta d_{50}\theta}}{u_{cr}} - 1} \tag{A.67}$$

4.4 Parker and Klingeman (1982)

The bed load transport formula for a gravel bed river can be expressed as

$$s_b = \frac{w^*}{\Delta}\ g^{1/2}\ (hI)^{3/2} \tag{A.68}$$

The expression for the parameter w^* is given as follows

$$w^* = \alpha_2\ (1 - \frac{0.822}{\phi_{50}})^{4.5} \quad for \quad \phi_{50} \rangle 1.65 \tag{A.69}$$

Substituting the expression w^* in equation (A.68) and then simplifying yields

$$s_b = A\ (1 - \frac{0.822}{\phi_{50}})^{4.5}\ u^3 \tag{A.70}$$

where A is considered constant for the sake of simplification of the derivation. A is given by the following expression:

$$A = \frac{\alpha_2 g^{1/2}}{\Delta C^3} \tag{A.71}$$

Now, differentiating equation (A.70) with respect to 'u' gives

$$\frac{ds_b}{du} = A \left[7.398 \frac{u^2}{\phi_{50}} \left(1 - \frac{0.822}{\phi_{50}}\right)^{3.5} + 3u^2 \left(1 - \frac{0.822}{\phi_{50}}\right)^{4.5}\right] \tag{A.72}$$

Applying equation (A.3) in equation (A.72) yields

$$A \left[7.398 \frac{u^2}{\phi_{50}} \left(1 - \frac{0.822}{\phi_{50}}\right)^{3.5} + 3u^2 \left(1 - \frac{0.822}{\phi_{50}}\right)^{4.5}\right] = \frac{n}{u} s_b \tag{A.73}$$

By combining equations (A.70) and (A.73), it is possible to obtain the exponent 'n' as

$$n = 3 \left[\frac{\phi_{50} + 1.644}{\phi_{50} - 0.822}\right] \tag{A.74}$$

where ϕ_{50} is equal to $\tau_{50}^{*}/0.0876$ and τ_{50}^{*} is equal to $hS/\Delta d_{50}$. The value of α_2 is equal to 11.2, according to Parker and Klingeman.