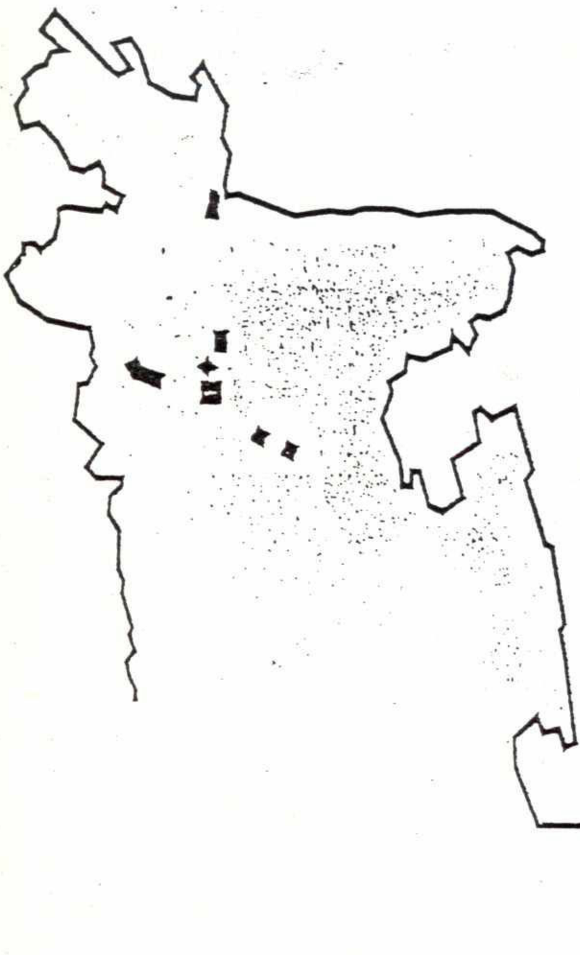
GOVERNMENT OF BANGLADESH

FLOOD PLAN COORDINATION ORGANIZATION

FAP 24 RIVER SURVEY PROJECT

RIVER HYDRAULICS



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DANISH HYDRAULIC INSTITUTE
OSIRIS
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Project ALA/90/04 — Commission of the European Communities

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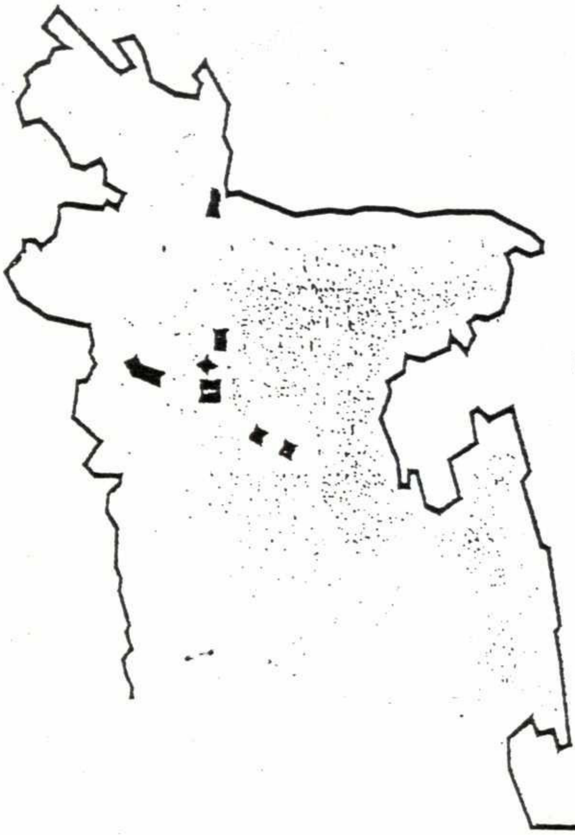

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LECTURE NOTES ON RIVER HYDRAULICS BY H.N.C. BREUSERS

DELFT
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Saleem Mahmood and Pieter van Groen.

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LIST OF SYMBOLS (HYDRAULICS)

		Dimension
A	= surface area of cross section	m^2
A_c	= surface area of flow-carrying part of cross section	m^2
B	= surface (storage) width	m
B_c	= width of flow-carrying part of cross section	m
C	= Chézy constant	$m^{\frac{1}{2}}/s$
c	= celerity	m/s
D	= pipe diameter	m
Fr	= Froude number	-
g_x	= component of g in the x-direction	m/s^2
g	= gravitational constant	m/s^2
H	= energy level	m
h	= water depth	m
h_c	= critical depth	m
h_n	= equilibrium (normal) depth	m
I	= energy gradient	-
i_b	= bed level slope	-
K	= compressibility	N/m^2
k_s	= Nikuradse roughness	m
n	= Mannings coefficient	-
P	= wetted perimeter	m
p	= pressure	N/m^2
Q	= discharge	m^3/s
q	= discharge /m width	m^2/s
R	= hydraulic radius, radius of bend	m
r	= radial coordinate	m
U,V,W	= velocities in x,y,z direction	m/s
u'	= velocity fluctuation	m/s
$\widetilde{U}$	= time averaged velocity	m/s
$\bar{U}$	= depth-averaged velocity	m/s
σ_u	= r.m.s. value of u'	m/s
u^*	= shear velocity	m/s
x,y,z	= coordinate in flow, lateral and vertical direction	m
z_b, z_w	= vertical coordinate of bed and water surface	m
α'	= coefficient due to non-uniformity of velocities	-
δ	= thickness of viscous sublayer	m
ε_m	= eddy viscosity	m^2/s
η	= dynamic viscosity	Ns/m^2
κ	= von Kármán constant	-

λ	= Darcy - Weisbach friction factor	-
ν	= kinematic viscosity	m^2/s
ξ	= loss coefficient	-
ρ	= density	kg/m^3
σ	= surface tension	N/m'

1. INTRODUCTION

Rivers form complicated systems, involving the motion of water and sediments and their mutual interaction. Flowing water transports sediment, the transport causes changes in the river bed and banks which influences the water motion again. The time scales of the two motions are different. Flood waves in rivers have time scales in the order of days whereas changes in a river bed take place over periods of years. In many cases the two processes can be separated therefore to study their specific properties, but to predict river bed changes the two phenomena have to be considered together.

Prediction of river behaviour is important in view of the many uses of rivers: flood control, navigation, waterpower, water supply, sand and gravel supply etc. Each use has some influence on river behaviour which has to be predicted.

Rivers have a great variety in size with the Amazone River in Brasil as the largest by far. The cross section at Obidos, 800 km from the mouth shows a width of 2300 m, a maximum depth of 60 m and the maximum discharge measured here in 1953 was 280,000 m^3/s with velocities in the order of 2.0 m/s. The world champion in sediment transport is the Hwang Ho or Yellow River, transporting 2,000,000,000 tons of sediment per year.

The purpose of this course is to review those aspects of hydraulics and sediment transport which are necessary in the analysis of alluvial river systems. The course has four parts:

1. Basic hydraulics
2. Sediment transport
3. Rivers
4. Modelling.

This part discusses some aspects of hydraulics such as steady and unsteady flow, uniform and nonuniform flow and the flow over and in structures. The course presents only an introduction. For more information reference is made to handbooks and literature.

2. PROPERTIES OF WATER. FLOW TYPES

Some of the relevant properties of water are:

Property	symbol	dimension	remarks
density	ρ	(kg/m^3)	
dynamic viscosity	η	$(kg/m.s)$ or $(N.s/m^2)$	$\tau = \eta \frac{\partial U}{\partial z}$
kinematic viscosity	ν	(m^2/s)	$\nu = \eta/\rho$
surface tension	σ	(kg/s^2) or (N/m)	
compressibility	K	$(kg/m.s^2)$ or N/m^2	

The following S.I. units are used:

mass	(kg)	basic units in the kg-m-s system	(kilogram)
length	(m)		(meter)
time	(s)		(second)
force	(kgm/s^2) or (N)		(Newton)
energy	(kgm^2/s^2) or (Nm) or (J)		(Joule)
power	(kgm^2/s^3) or (Nm/s) or (J/s) or (W)		(Watt)
pressure, stress	(kg/ms^2) or (N/m^2) or Pa		(Pascal)

2.1. Density (kg/m^3)

The density of fresh water varies with temperature T:

T:	0	4	12	16	21	32	$(^\circ C)$
ρ:	999.87	1000.0	999.5	999.0	998.0	995.0	(kg/m^3)

The variation of the density may be neglected in sediment transport calculations and river hydraulics.
Density differences, caused by salinity, are important in estuaries.

	kg/m^3
ρ fresh water	1000
ρ sea water	1026

2.2. Gravitational acceleration g (m/s^2)

Depends on latitude α:

equator	($\alpha = 0^o$)	$g = 9.780$ m/s^2	at mean sea level
poles	($\alpha = 90^o$)	$g = 9.832$ m/s^2	
Netherlands	($\alpha = 52^o$)	$g = 9.813$ m/s^2	

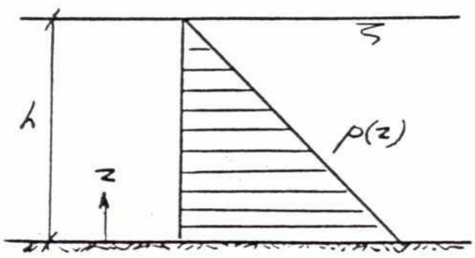
2.3. Pressure p (N/m^2)

Pressure (p) is isotropic (independent of direction) in fluids at rest (Pascal's law).
The pressure is hydrostatic in this condition:

$$\frac{\partial p}{\partial z} = - \rho g$$

In fluids with a free surface (p relative to the atmospheric pressure):

$$p = \rho g \ (h-z)$$



2.4. Compressibility K (N/m^2)

The liquid bulk modulus of elasticity is defined by:

$$K = \frac{-dp}{dV/V}$$

dp = pressure change

dV = volume change

Compressibility can be neglected in free surface flows.
(not in pipe flow: water hammer).

$$K \simeq 2.10^9 \text{ N/m}^2 \quad \text{for water at } 0^oC.$$

2.5. Viscosity

Dynamic viscosity (Ns/m^2)

Defined as the factor of proportionality in:

$$\tau = \eta \frac{\partial U}{\partial z}$$

which is valid for laminar flow.

$\partial U/\partial z$ = velocity gradient (s^{-1}) τ = shear stress (N/m^2)

η = constant for a Newtonian fluid.

η = 0 in an "ideal" fluid.

Kinematic viscosity (m^2/s)

Defined by $\nu = \eta/\rho$

η and ν are a function of temperature. The influence of temperature is significant.

T	0	5	10	15	20	25	30	35	40	($^\circ C$)
ν	1.79	1.52	1.31	1.14	1.01	0.90	0.80	0.72	0.65	(10^{-6} m^2/s)

2.6. Surface tension σ (N/m)

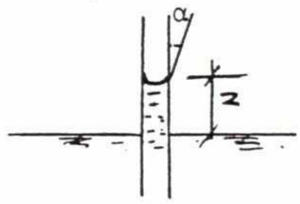
For the surface water/air: σ = 0.074 N/m at atmospheric pressure.

The variation with temperature can be neglected.

Effect: capillary rise.

For a tube with diameter D; filled with water.

Contact angle α = 0.



$$z = \frac{4\sigma}{\rho g D} = \frac{4.0.074}{10^4.D} \simeq \frac{3.10^{-5}}{D} \text{ m}$$

D = 1 mm → z = 30 mm D = 0.1 mm → z = 300 mm

FLOW TYPES

2.7. Laminar - turbulent flow

Reynolds observed that a coloured fluid was not laterally dispersed in pipe flow for sufficiently low velocities. Increasing the velocity gave a rather sudden transition to a condition with very intensive mixing. The first condition is called <u>laminar</u> flow, the second <u>turbulent</u> flow.

Transition occurs roughly at a (Reynolds)number based on average velocity, pipe diameter and viscosity:

$$Re = \frac{\bar{U}D}{\nu} \simeq 2000$$

For an open channel with depth h transition occurs roughly at

$$Re = \frac{\bar{U}h}{\nu} \simeq 600$$

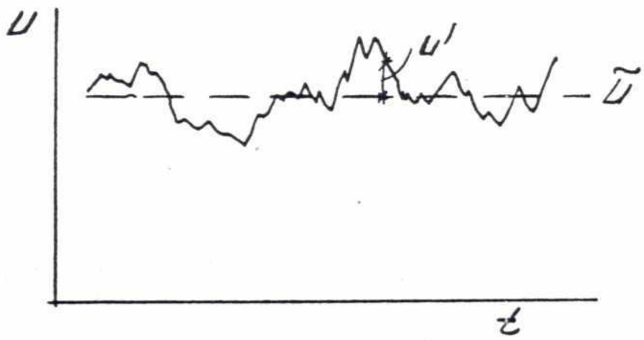
In river engineering practice only turbulent flows are of importance. (take $h = 1$ m $\quad \bar{U} = 1$ m/s $\quad \nu = 10^{-6} m^2/s \quad$ or $\quad Re = 10^6$).
The velocity in a point in a turbulent fluid fluctuates with time:

$$U(t) = \tilde{U} + u'(t)$$

$$\tilde{U} = \frac{1}{T}\int_0^T U(t)\, dt$$

If T is sufficiently large:

$$\tilde{u}' = 0$$



The intensity of the turbulence is characterised by the root-mean square (r.m.s.) value:

$$\sigma_u = \left[\frac{1}{T}\int_0^T (u')^2 dt\right]^{\frac{1}{2}}$$

or the relative turbulence intensity:

$$r = \frac{\sigma_u}{\bar{U}}$$

r varies from a few percent to 30 - 40 % in very turbulent conditions.

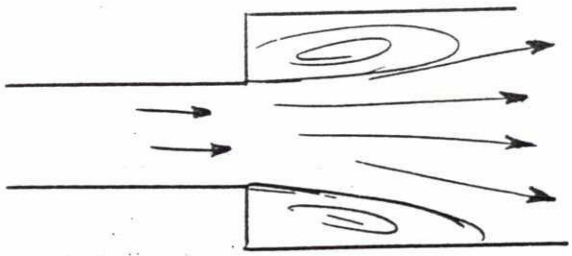
2.8. Steady - unsteady flow

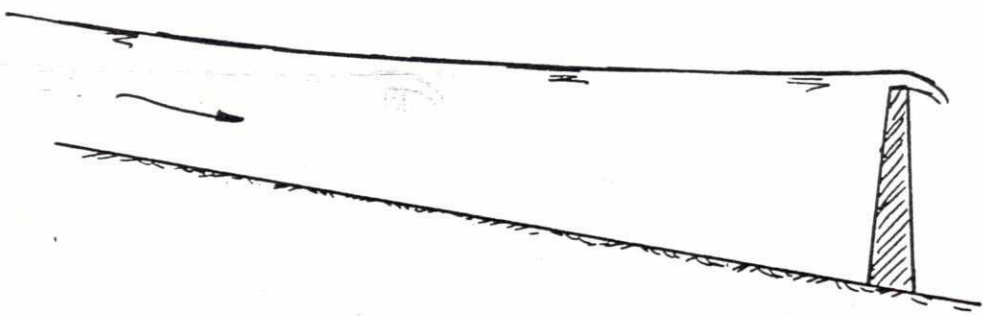
Steady flow $\quad \tilde{U}$ independent of time.

Unsteady flow $\quad \tilde{U}$ dependent of time (flood waves, tides, water hammer).

2.9. Uniform - non-uniform flow

Uniform flow: independent of coordinate in flow direction. Examples of non-uniform flow: flow in expansions, back-water curves.

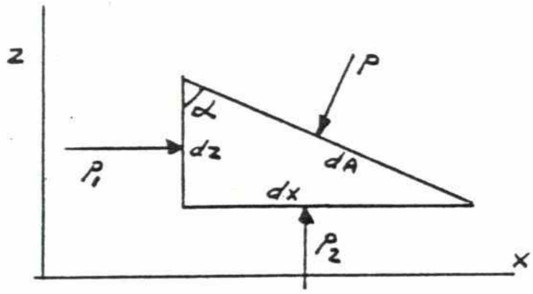




3. HYDROSTATICS

3.1. Law of Pascal. Hydrostatic pressure distribution

It can be shown that pressure is isotropic (independent of direction) in a fluid at rest (law of Pascal).
For a two-dimension situation it can be demonstrated as follows:



Neglecting the weight of the element (second-order effect) the following equations are valid:
Horizontal equilibrium:

$$p dA \cos\alpha = p_1 dz = p_1(dA \cos\alpha)$$

$$\text{or } p = p_1$$

Vertical equilibrium:

$$p dA \sin\alpha = p_2 dx = p_2(dA \sin\alpha)$$

$$\text{or } p = p_2$$

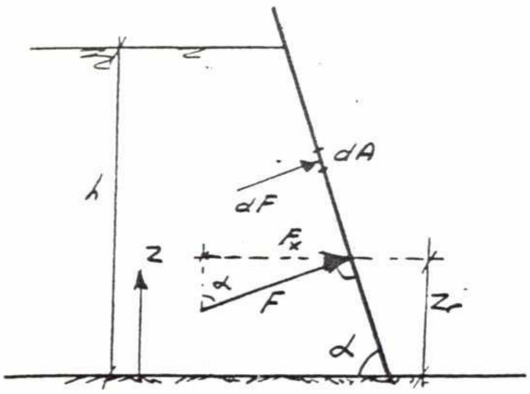
Therefore the pressure is isotropic: $p = p_1 = p_2$
The proof can be easily extended to a three-dimensional element. Pascal's law also holds to a sufficient degree of approximation for a moving fluid if pressure gradients are large in comparison to shear stress gradients.

The pressure decreases linearly with the vertically upward direction:

$$\frac{\partial p}{\partial z} = -\rho g \qquad (3.1)$$

For free-surface flows, taking the atmospheric pressure as a reference level: $p = \rho g(h-z)$.
This equation also holds in flowing fluids if the streamlines are not curved in the vertical plane and vertical accelerations may be neglected.

3.2. Pressure on a wall



The pressure at each depth is given by:

$$p = \rho g(h-z) \qquad (3.2)$$

The force on an element dA (unit width) is equal to:

$$dF = p dA = \rho g(h-z)\frac{dz}{\sin\alpha}$$

Integration gives:

$$F = \int_0^h \rho g(h-z)\frac{dz}{\sin\alpha} = \frac{1}{2}\rho g h^2 \frac{1}{\sin\alpha}$$

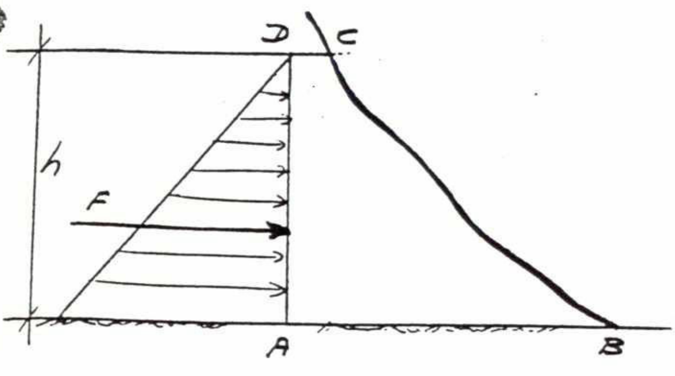
The action point of this force is at 1/3 of the depth :

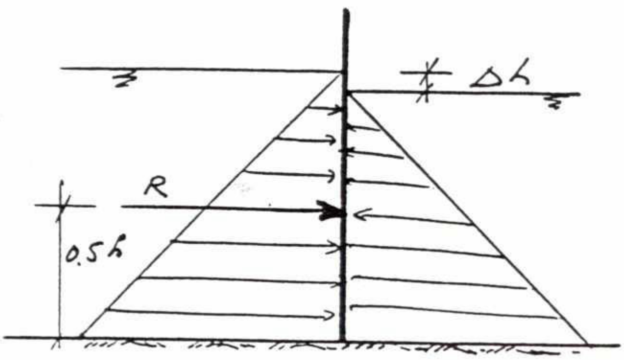
$$z_r = \frac{1}{3}h$$

The horizontal component of this force is equal to:

$$F_x = F\sin\alpha = \frac{1}{2}\rho g h^2 \qquad (N/m') \qquad (3.3)$$

This force is independent of α and even independent of the shape of the wall (consider the horizontal equilibrium of ABCD.



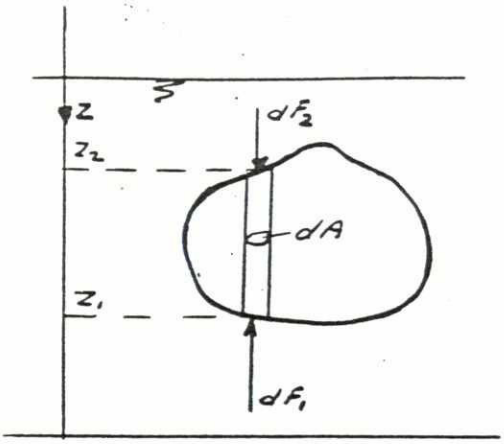


The resultant force on a wall due to a small water-level difference Δh is equal to $R = \rho g \Delta h$ (N/m') (3.4)

with an action point at mid depth.

3.3. Buoyancy

An object in a fluid experiences an upward force due to the pressure differences on its surface.



The net vertical force on a cylinder with cross section dA is equal to:

$$dF = dF_1 - dF_2 = \rho g z_1 dA - \rho g z_2 dA$$

$$= \rho g dA \; (z_1 - z_2)$$

Integration over the surface of the object gives:

$$F = \int_A \rho g dA(z_1 - z_2) = \rho g \int_A dA(z_1 - z_2) = \rho g \cdot \text{volume} \tag{3.5}$$

The net vertical force is equal to the weight of the displaced volume (law of Archimedes).

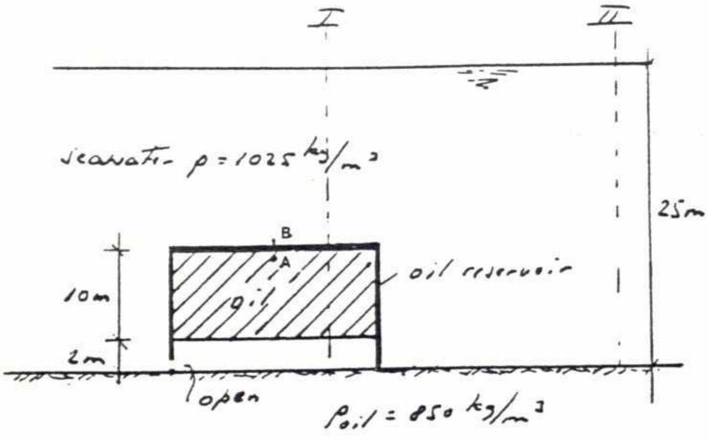
It can be shown that the action line of this force runs through the centre of gravity of the volume. The object will sink or float depending on:

$G \gtrless F$ $\qquad$ G = weight of the object.

or

$\rho_{object} \gtrless \rho_{water}$

3.5

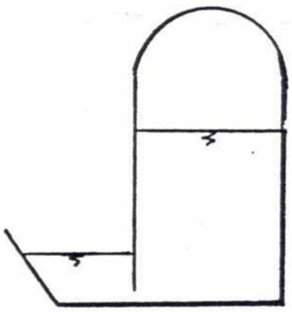


Is p_A larger or smaller than p_B ?
p = pressure

Draw the pressure distribution over the vertical inside and outside the reservoir (sections I and II).

Compute $p_A - p_B$

3.6



Explain the functioning of a water reservoir for birds.

4. BASIC EQUATIONS

4.1. Introduction

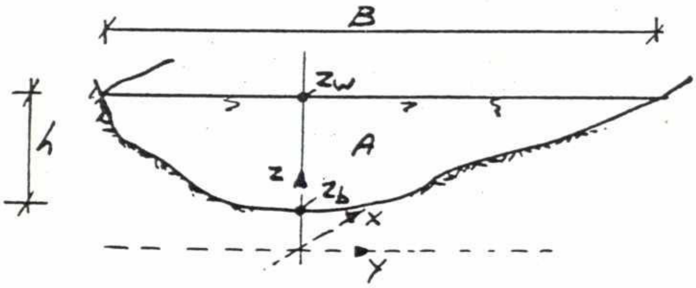
The motion of water can be described by considering continuity and the equations of motion. In some cases also the energy equation can be useful. In many cases equations can be simplified by averaging over depth and/or width. This is expecially useful in river problems, where in many cases only depth-averaged or profile-averaged quantities are of importance.

4.2. Continuity

Considering the mass balance for a unit volume, assuming constant density, leads to the continuity equation:

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} = 0 \qquad (4.1)$$

This equation is also valid in turbulent flow. In that case the velocities denote time-averaged values.



Integration of (4.1) over the depth leads to the equation:

$$\frac{\partial z_w}{\partial t} + \frac{\partial}{\partial x}(h\bar{U}) + \frac{\partial}{\partial y}(h\bar{V}) = 0 \qquad (4.2)$$

in which $\bar{U}$ and $\bar{V}$ are the depth-averaged velocities. Averaging over the width gives:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \qquad (4.3)$$

in which A = cross section Q = discharge.
In words: a change in discharge has to be stored by a change in depth.
Differentiating the first term gives:

$$B\,\frac{\partial z_w}{\partial t} + \frac{\partial Q}{\partial x} = 0 \qquad (4.4)$$

in which B = the width at the water surface (the storage width). For a truly two-dimensional flow Eq. (4.4) reduces to:

$$\frac{\partial z_w}{\partial t} + \frac{\partial q}{\partial x} = 0 \tag{4.5}$$

q = discharge / unit width.

4.3. Equations of motion

The equations of motion are based on Newton's second law:

$$\vec{F} = m.\vec{a} = m.\frac{d\vec{U}}{dt} \tag{4.6}$$

The forces acting are : - pressure gradients
- shear stress gradients
- gravity
- wind etc.

$d\vec{U}$ is the total differential. Consider the component in the x-direction:

$$dU = \frac{\partial U}{\partial t}.dt + \frac{\partial U}{\partial x}.dx + \frac{\partial U}{\partial y}.dy + \frac{\partial U}{\partial z}.dz \qquad U = U(x,y,z,t)$$

or
$$\frac{dU}{dt} = \frac{\partial U}{\partial t} + \frac{\partial U}{\partial x}.U + \frac{\partial U}{\partial y}.V + \frac{\partial U}{\partial z}.W \tag{4.7}$$

$$(\frac{dx}{dt} = U \text{ etc.}).$$

The pressure gradients give forces per unit volume of the form:

$$\frac{\partial p}{\partial x}, \frac{\partial p}{\partial y}, \frac{\partial p}{\partial z}$$

Viscous shear stresses give contribution of the form:

$\eta \frac{\partial^2 U}{\partial x^2}$ etc. but these can be neglected in general for turbulent flows.

The equation of motion (as given by Euler) become:

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} + W\frac{\partial U}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + g_x \tag{4.8}$$

$$\frac{\partial V}{\partial t} + U\frac{\partial V}{\partial x} + V\frac{\partial V}{\partial y} + W\frac{\partial V}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + g_y \tag{4.9}$$

$$\frac{\partial W}{\partial t} + U\frac{\partial W}{\partial x} + V\frac{\partial W}{\partial y} + W\frac{\partial W}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + g_z \tag{4.10}$$

g_x is the component of g in the x direction.

Time-averaging the equations (4.13) and (4.14) leads to:

$$\frac{\partial \overline{U}}{\partial t} + \frac{\partial}{\partial x}\overline{U^2} + \frac{\partial}{\partial y}(\overline{U.V}) + \frac{\partial}{\partial z}(\overline{U.W}) + g\frac{\partial z_w}{\partial x} - \frac{1}{\rho}\frac{\partial \tau_{xz}}{\partial z} = 0 \qquad (4.15)$$

$$\frac{\partial \overline{V}}{\partial t} + \frac{\partial}{\partial x}(\overline{U.V}) + \frac{\partial}{\partial y}\overline{V^2} + \frac{\partial}{\partial z}(\overline{V.W}) + g\frac{\partial z_w}{\partial y} - \frac{1}{\rho}\frac{\partial \tau_{yz}}{\partial z} = 0 \qquad (4.16)$$

in which $\tau_{xz} = -\rho\overline{u'w'} + \mu\frac{\partial \overline{U}}{\partial z}$ (4.17)

$$\tau_{yz} = -\rho\overline{v'w'} + \mu\frac{\partial \overline{V}}{\partial z} \qquad (4.18)$$

(The viscous shear stresses have been shown, to demonstrate that the terms resulting from averaging (UV) etc. have the character of a shear stress. They are called turbulent shear stresses or Reynold's stresses. In fact, they are not shear stresses, but represent the effect of the transport of momentum by the turbulence. Gradients of $\overline{u'^2}$, $\overline{u'v'}$ and $\overline{v'^2}$ can be neglected in general).
To solve the equations (4.15) and (4.16) also boundary conditions have to be given. At the free surface $\tau_{xz} = \tau_{yz} = 0$ (assuming no wind shear stress) and at the bottom also a shear stress has to be specified as will be discussed later.

For river problems averaging over the depth is generally applied. This leads to:

$$\frac{\partial}{\partial t}(h.\overline{U}) + \frac{\partial}{\partial x}(\alpha_1 h\overline{U}^2) + \frac{\partial}{\partial y}(\alpha_2 h\overline{U}.\overline{V}) + gh\frac{\partial z_w}{\partial x} + \frac{1}{\rho}\tau_{xb} = 0 \qquad (4.19)$$

$$\frac{\partial}{\partial t}(h.\overline{V}) + \frac{\partial}{\partial x}(\alpha_2 h\overline{U}.\overline{V}) + \frac{\partial}{\partial y}(\alpha_3 h\overline{V}^2) + gh\frac{\partial z_w}{\partial y} + \frac{1}{\rho}\tau_{yb} = 0 \qquad (4.20)$$

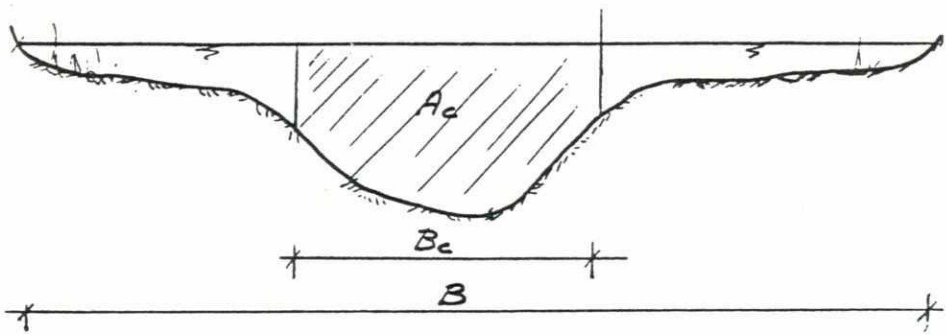
$\overline{U}$ and $\overline{V}$ indicate the depth-averaged velocities.
τ_{xb} is the component of the bed shear stress in the x-direction.
$\alpha_{1,2,3}$ represent the effect of the non-uniformity in the velocities with depth, for example:

$$\alpha_1 = \frac{1}{h.\overline{U}^2}\cdot\int_{z_b}^{z_w} U^2 dz \qquad (4.21)$$

To obtain cross-sectional averages, a second integration is carried out over the width. The result is (with some approximation, see for example Jansen 1979):

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\,\alpha^1\,\frac{Q^2}{A} + gA\,\frac{\partial z_w}{\partial x} + \frac{P.\tau_b}{\rho} = 0 \qquad (4.22)$$

where Q is the discharge and A is the cross-sectional area. α^1 expresses the non-uniformity of the velocity distribution, P is the wetted perimeter and τ_b the average bed shearstress. α^1 can be computed if the velocity distribution is known. It is more convenient for complicated cross sections to define a conveying cross section A_c with almost uniform velocity so that $\alpha^1 \simeq 1.0$.



Because the rest of the cross section does not contribute to the momentum transport, A may be replaced by A_c in Eq. 4.22. Neglecting variations of A_c and α^1 with x, the second term in Eq. (4.22), using the continuity equation (4.3), becomes:

$$\frac{\partial}{\partial x}\left(\frac{Q^2}{A_c}\right) = \frac{2\,Q}{A_c}\cdot\frac{\partial Q}{\partial x} - \frac{Q^2}{A_c{}^2}\cdot\frac{\partial A_c}{\partial x} = -\,\frac{2Q}{A_c}\cdot\frac{\partial A}{\partial t} - \frac{Q^2}{A_c{}^2}\left(B_c\,\frac{\partial h}{\partial x} + \cancel{\frac{\partial A}{\partial x}}\Big|_{h=c}\right)$$

so that Eq. (4.23) becomes:

$$\frac{\partial Q}{\partial t} - \alpha^1\,\frac{2Q}{A_c}\cdot\frac{\partial A}{\partial t} + gA_c\left(1-\alpha^1\,\frac{Q^2 B_c}{gA_c{}^3}\right)\frac{\partial h}{\partial x} + gA_c\,\frac{\partial z_b}{\partial x} + \frac{P\tau_b}{\rho} = 0 \qquad (4.23)$$

where $z_b = z_w - h$ is the bottom level in the conveying cross section.

In a wide cross section with uniform depth h, Eq. (4.22) reduces to:

$$\frac{\partial \bar{U}}{\partial t} + \alpha^1\,\bar{U}\,\frac{\partial \bar{U}}{\partial x} + g\,\frac{\partial z_w}{\partial x} + \frac{\tau_b}{\rho h} = 0 \qquad (4.24)$$

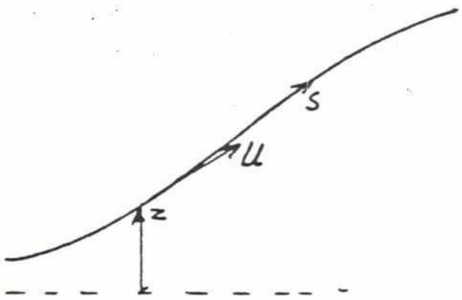
The second term may be neglected in cases where:

$\frac{\bar{U}^2}{gh} << 1$ (compare the second and the third term and take

$\frac{\partial z_w}{\partial x} = \frac{\partial h}{\partial x}$, neglecting the bed-level change).

4.4. Energy equation (Bernoulli's equation)

The equation of motion (4.11) also holds along a stream line s in steady flow $(\frac{\partial U}{\partial t} = 0)$ neglecting friction.



Because s is a stream line there is only a flow velocity in the flow direction s, therefore (4.11) reduces to:

$$U \frac{\partial U}{\partial s} = - \frac{1}{\rho} \frac{\partial p}{\partial s} + g_s \qquad (4.25)$$

This may be integrated to give:

$$\frac{1}{2} U^2 + \frac{p}{\rho} + gz = \text{constant (along a stream line)}$$

or $$\frac{p}{\rho g} + z + \frac{U^2}{2g} = H \qquad \text{the Bernoulli equation.} \qquad (4.26)$$

$\frac{p}{\rho g}$ = pressure head

z = position head

$\frac{U^2}{2g}$ = velocity head

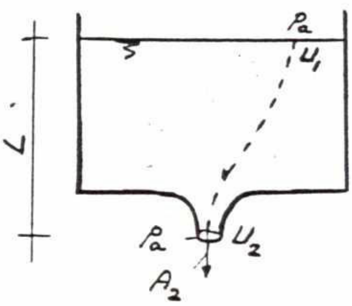
H = energy level

$\frac{p}{\rho g} + z$ = piezometric level

This equation can of course also be derived by considering conservation of energy (no friction). Friction can be neglected in strongly accelerating (with distance or time) flows.

Applications:

A1 Law of Torricelli.



If a constant level is maintained in a vessel with atmospheric pressure both at the surface and the discharge point, neglecting friction the Bernoulli equation (4.26) gives:

$$\frac{p_1}{\rho g} + z_1 + \frac{U_1^2}{2g} = H = \frac{p_2}{\rho g} + z_2 + \frac{U_2^2}{2g}$$

For a wide vessel $U_1 \simeq 0$. $p_1 = p_2 = p_a$ $z_1 - z_2 = L$

or $L = \frac{U_2^2}{2g}$ or $U_2 = \sqrt{2gL}$

The discharge is $Q = A_2 \cdot U_2 = A_2 \sqrt{2gL}$ (4.27)

Another application is the Pitot tube, used to measure velocities.

A2 Rapidly varied flow

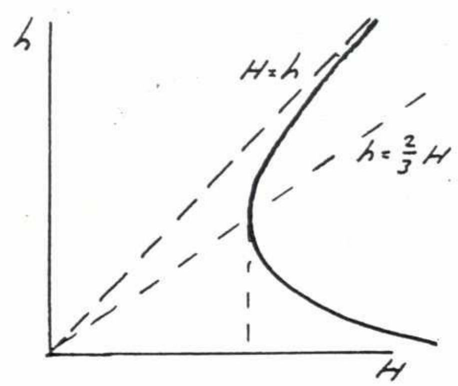
If flow is accelerated over a short distance, friction can be neglected in general. If also a uniform velocity distribution is assumed, then 4.26 holds for all flow lines or:

$$H = h + \frac{U^2}{2g} = \text{constant}$$

with $q = U.h$ this gives:

$$H = h + \frac{q^2}{2gh^2} \tag{4.28}$$

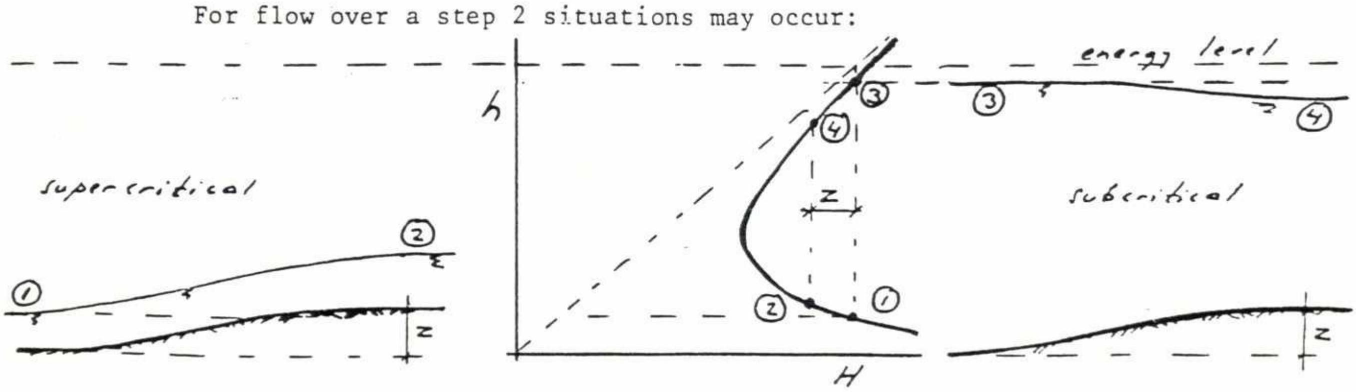
For a given H and q this is a third-order equation in h, which gives two positive roots for h in certain conditions. The minimum value of H for a given q is found from $dH/dh = 0$ which gives $h = h_c = 2/3H$. h_c is the critical depth at which for a given q water is transported with minimum H. For $h > h_c$ the flow is subcritical or tranquil, for $h < h_c$ supercritical or shooting.



For $h = 2/3H$ it follows that $\frac{U^2}{2g} = \frac{1}{3} H$ or $\frac{U}{\sqrt{gh}} = 1$ (Froude number)

The transition of subcritical and supercritical flow occurs at a Froude number of 1.

For flow over a step 2 situations may occur:



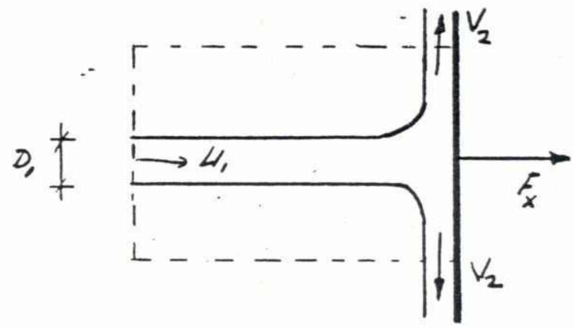
In both cases H has to be reduced with the increase in position head z to obtain the downstream water depths.
The subcritical case gives a decrease in water level, the supercritical case an increase.

4.5. Momentum equation

Newton's equation can be used in the form:

$$\vec{F}dt = d(m\vec{U})$$

and applied to a control volume. Consider steady flow in a fixed coordinate system (Euler) and take for the control volume a stream tube (a tube bounded by stream lines so that no fluid passes the boundary).
m is the mass flow considered or $m = \rho.Qdt$ in which Q is the discharge through the flow tube.
Taking the equation in the x-direction:

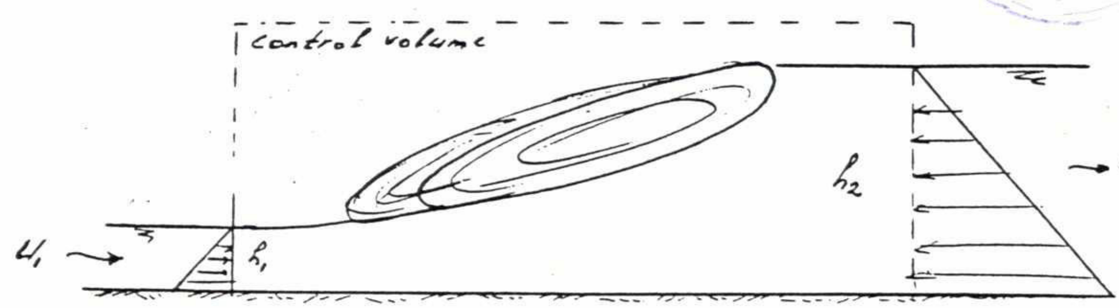


$$-F_x dt = \rho Qdt.\ dU$$
$$= \rho Qdt\ (U_2-U_1)$$

For a jet with velocity a U_1 acting on a plate, the resulting force on the plate is equal to ($U_2 = 0$):

$$F_x = \rho Q.U_1 \tag{4.29}$$
$$= \rho.\frac{1}{4}.\pi D_1{}^2.U_1{}^2$$

Application: The hydraulic jump



A transition from supercritical to subcritical flow generally gives a hydraulic jump. Application of momentum balance gives: (the forces are

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here the hydrostatic pressure forces:

$$\frac{1}{2}\rho g h_1{}^2 \quad \text{and} \quad \frac{1}{2}\rho g h_2{}^2\ ):$$

$$\frac{1}{2}\rho g h_1{}^2 - \frac{1}{2}\rho g h_2{}^2 = \rho q U_2 - \rho q U_1$$

Substitution of $U_2 = q/h_2$ and $U_1 = q/h_1$ and multiplication with $2h_1h_2/\rho g$ gives:

$$h_1h_2(h_1-h_2)(h_1+h_2) = \frac{2q^2}{g}(h_1-h_2)$$

The solution $h_1 = h_2$ is trivial (no motion)
therefore:

$$h_2{}^2 + h_1h_2 - \frac{2q^2}{gh_1} = 0$$

or $$h_2 = -\frac{1}{2}h_1 \pm \sqrt{\frac{1}{4}h_1{}^2 + 2q^2/gh_1} = \frac{1}{2}h_1\left[\sqrt{\frac{8q^2}{gh_1{}^3}+1} - 1\right] \qquad (4.30)$$

(the negative root has no physical relevance).

A solution is possible for $h_2 > h_1$ or

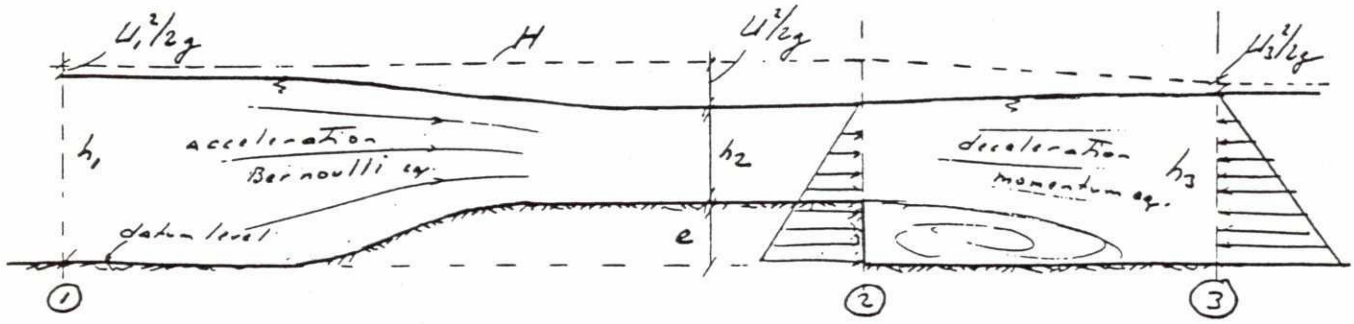
$$\sqrt{\frac{8q^2}{gh_1{}^3}+1} > 3$$

or $\dfrac{q^2}{gh_1{}^3} > 1$ or $Fr_1 = \dfrac{U_1}{\sqrt{gh_1}} > 1$

Substitution of the Fr_1 number gives:

$$\frac{h_2}{h_1} = \frac{1}{2}\left[\sqrt{8Fr_1{}^2+1} - 1\right]$$

<u>Application of Bernoulli and momentum equation</u>



From ① to ② we have an acceleration zône, so friction is neglected and Bernoulli's equation can be applied:

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$$z_1 + h_1 + \frac{U_1^2}{2g} = z_2 + h_2 + \frac{U_2^2}{2g}$$

$$0 + h_1 + \frac{U_1^2}{2g} = e + h_2 + \frac{U_2^2}{2g}$$

$$q = U_1 h_1 = U_2 h_2$$

$$h_1 + \frac{q^2}{h_1^2 . 2g} = e + h_2 + \frac{q^2}{h_2^2 . 2g}$$

from which h_2 can be computed for given h_1 and q.

From ② to ③ , a deceleration zône, there is a strong energy dissipation by turbulent shear stresses, so Bernoulli's equation can not be used, only the momentum equation is valid.

$$\frac{1}{2} \rho g \, (h_2 + e)^2 - \frac{1}{2} \rho g \, h_3^2 = \rho q U_3 - \rho q U_2$$

$$\frac{1}{2} \rho g \, (h_2 + e)^2 - \frac{1}{2} \rho g \, h_3^2 = \rho \frac{q^2}{h_3} - \rho \frac{q^2}{h_2}$$

q and h_2 are known, so that h_3 can be computed.
Section ② has been taken just downstream of the step and it has been assumed that the pressure distribution is hydrostatic there. Of course the pressure also acts on the back side of the step, so in the momentum equation $(h_2 + e)$ is used to compute the pressure forces.

4.6. Correction factors for momentum and kinetic energy

In par. 4.3 and 4.4 it has been assumed that the distribution of the velocity was uniform. For non-uniform flow a correction factor α' has to be used in the momentum flux term:

$$\alpha' = \frac{\int^A U^2 dA}{\bar{U}^2 . A} \qquad \text{Boussinesq coefficient}$$

Because integration of the Bernoulli equation over a cross section leads to a term with U^3 (the transport of kinetic energy), a correction factor α has to be introduced

$$\alpha = \frac{\int^A U^3 dA}{\bar{U}^3 . A} \qquad \text{Coriolis coefficient}$$

Generally $\alpha > \alpha'$

In the formula of the critical depth:

$$\frac{U_c^2}{gh_c} = \frac{q^2}{gh_c^3} = 1$$

or $$h_c = \sqrt[3]{\frac{q^2}{g}}$$

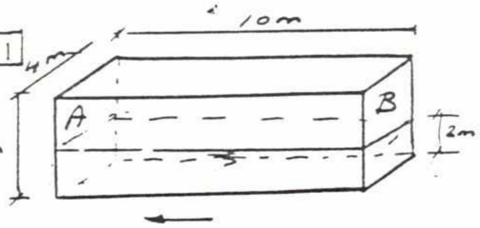
also the correction α has to be used for a non-uniform velocity distribution:

$$h_c = \sqrt[3]{\frac{\alpha q^2}{g}}$$

4.7 Problems

$\rho = 1000\ kg/m^3 \qquad g = 10\ ^{m}/s^2$

4.1



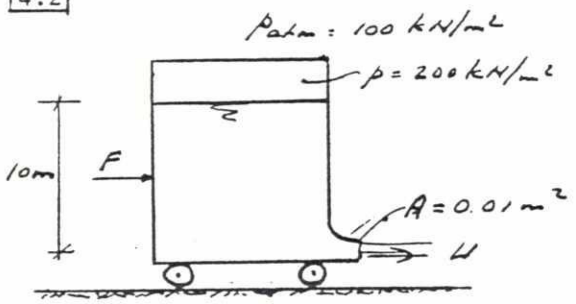
The basin, filled up to a depth of 2 m with water (at rest) is accelerated with an acceleration of 3 $^{m}/s^2$ in a horizontal direction.

a) What will be the waterdepths at sides A and B?

b) What will be the horizontal pressure forces on sides A and B?

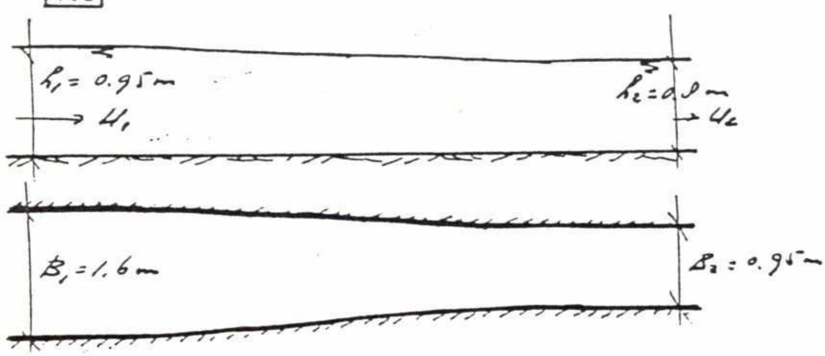
c) Show that the resultant force is equal to mass × acceleration for the water.

4.2



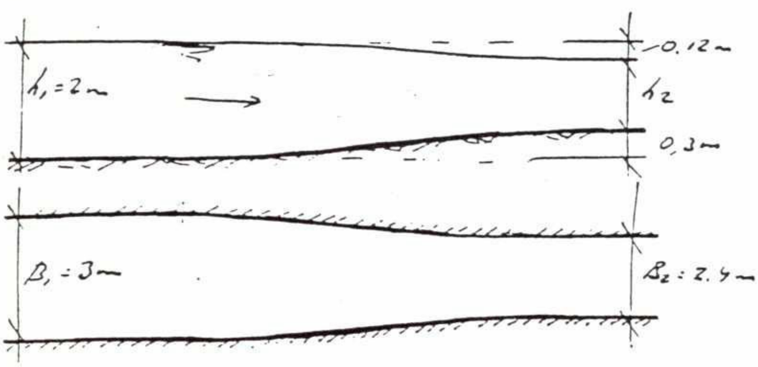
Water flows from a vessel without energy losses. How large is $\bar{U}$?
If the vessel is mounted on frictionless wheels, how large is the force F to keep it in position?

4.3



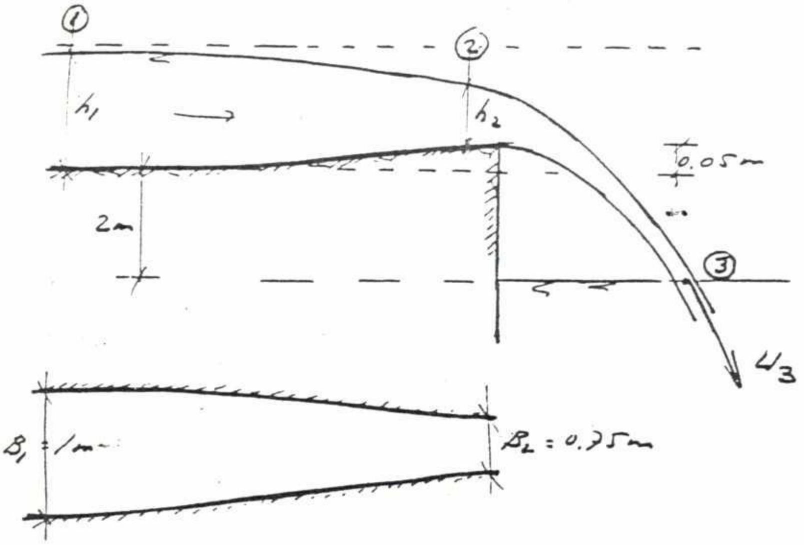
A channel is narrowed. The waterdepths are given. Compute the discharge Q assuming that energy losses can be neglected.

4.4



Same question as 4.3. Now with a rise in the bed level of 0,3 m.

4.5

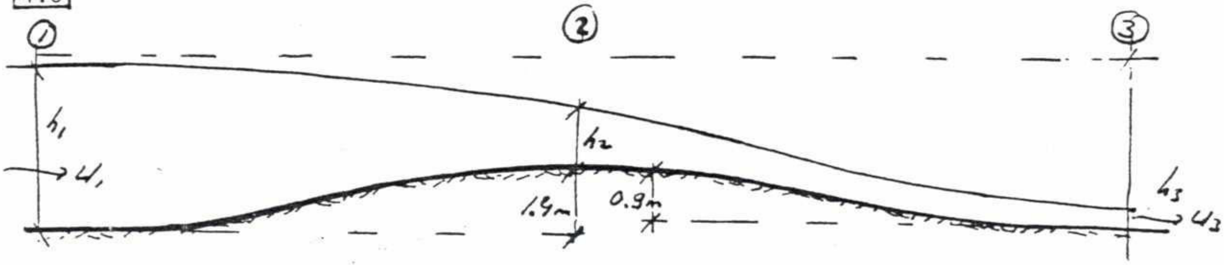


h_2 = 0,4 m

No energy losses

The flow in section 2 is critical. Compute Q, h_1 and U_3.

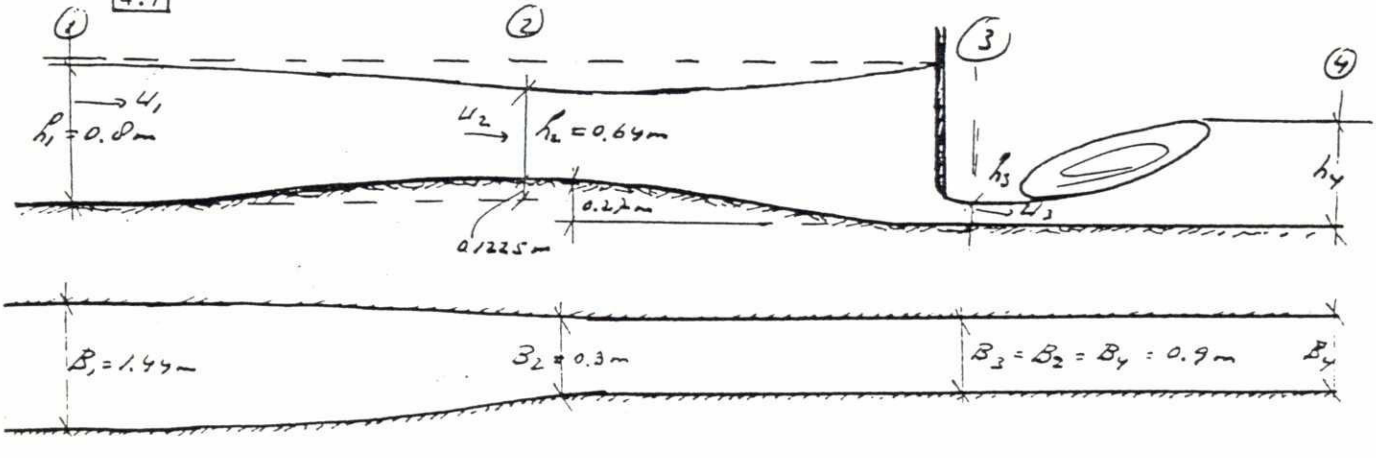
4.6



Constant channel width. No energy losses.

h_2 = 0,9 m. Compute q, h_1, U_1, h_3, U_3.

4.7



No energy losses between sections 1 and 3.

Compute Q, U_3, h_3, h_4, energy loss between 3 and 4.

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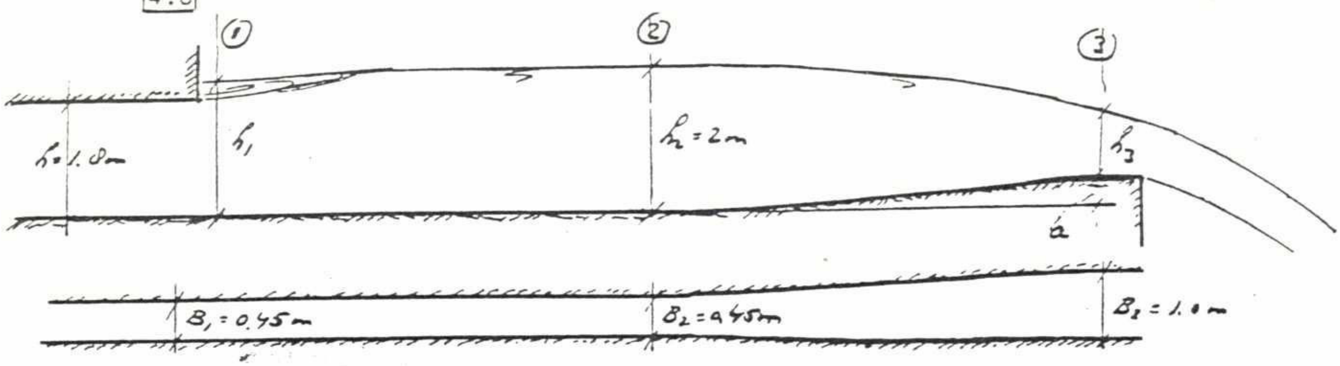
4.8

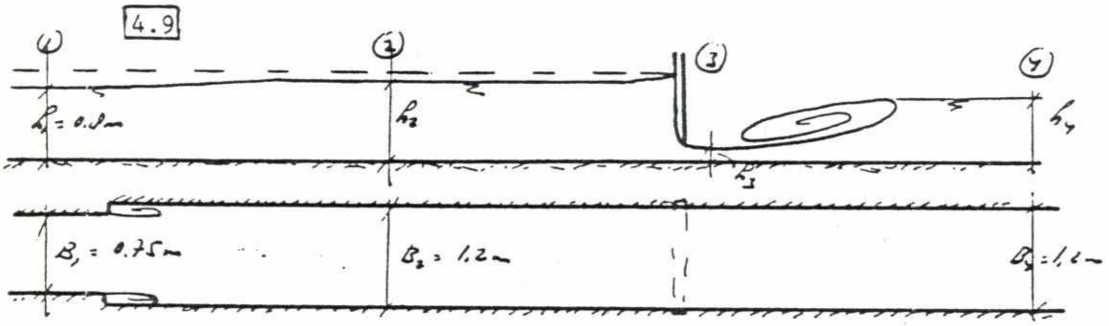


Water flows from a culvert with h = 1.8 m, B = 0.45 m in a basin with h_2 = 2 m. Q = 2.7 m^3/s.

a) Compute h_1, just downstream from the outlet.

b) How large must a be? (critical flow in section 3).

4.9



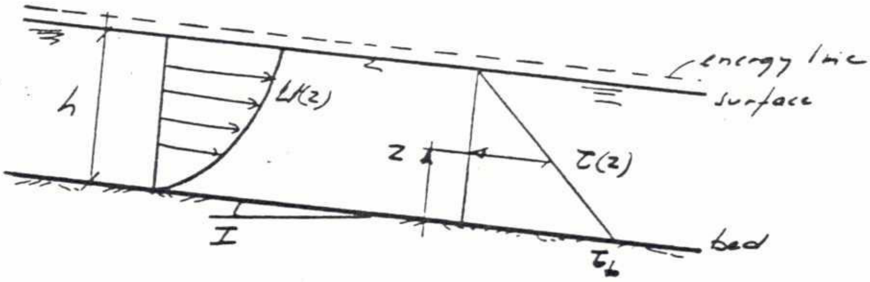
Q = 1.2 m^3/s

Compute h_2, h_3, h_4 and the energy loss between sections 3 and 4.
Neglect energy losses between sections 2 and 3.

5. UNIFORM FLOW IN OPEN CHANNELS AND PIPES

5.1. Equation of motion for steady uniform open channel flow

For a steady uniform flow ($\frac{\partial}{\partial t} = 0$, $\frac{\partial}{\partial x} = 0$, $V = 0$),



the depth-averaged equations of motion (4.24) reduces to:

$$g \frac{\partial z_w}{\partial x} + \frac{\tau_b}{\rho h} = 0$$

or $\tau_b = \rho g h I$ (sin I ≃ I for small I) (5.1)

in which I is the slope of the bed (and of water surface and energy line). Similarly it can be shown from the full equations of motion (4.15) that:

$$\tau(z) = \rho g\ (h-z)\ I = \tau_b \frac{h-z}{h} \qquad (5.2)$$

(the linear shear-stress distribution).

The shear-stress is equal to:

$$\tau(z) = \underbrace{-\rho\overline{u'w'}}_{\text{turbulent shear stress}} + \underbrace{\mu \frac{dU}{dz}}_{\text{viscous shear stress}}$$

The viscous shear stress is only important in laminar flow and in a thin layer close to a smooth wall in turbulent flow.

5.2. Velocity distribution

The difficulty is now the relation between shear stress and velocity distribution which is necessary to predict this distribution.

For laminar flow the relation is:

$$\tau(z) = \eta . \frac{\partial U(z)}{\partial z}$$

which leads to the parabolic velocity distribution:

$$U(z) = \frac{gI}{2\nu} (h^2 - (h - z)^2) \qquad (5.3)$$

and a mean velocity $\bar{U} = \frac{gI}{3\nu} . h^2$

Laminar flow only occurs for $Re = \frac{\bar{U}.h}{\nu} < 600$.

For turbulent flow Prandtl gave the following empirical mixing-length expression:

$$\tau(z) = -\rho\overline{u'w'} = \rho\ell^2 (\partial U(z)/\partial z)^2 \qquad \ell = \text{mixing length} \qquad (5.4)$$

Near the bed $\tau(z) \approx \tau_b$, the bed shear stress:

$$\tau_b = \rho gh\ I$$

and $\ell = \kappa z$

κ = kappa, von Kārmān's constant ≈ 0.4 (from measurements)

This leads to the logarithmic velocity distribution:

$$U(z) = \kappa^{-1} \sqrt{gh\ I} . \ln(z/z_o)$$

Define $u^* = \sqrt{gh\ I}$ = shear velocity = $\sqrt{\tau_b/\rho}$

and take: $\kappa = 0.4$

then: $U(z) = 2.5\ u^* \ln (z/z_o)$ (5.5)

z_o = the point where $U = 0$ according to the logarithmic profile.

$U(z)$ is equal to the mean velocity at $z \approx 0.4\ h$ (in fact at $z = \frac{1}{e} h$)

or $\bar{U} = 2.5\ u^* \ln (0.4\ h/z_o)$

or $\bar{U} = 5.75\ u^* \log (0.4\ h/z_o)$ (ln $\rightarrow$ log gives factor 2.303)

Although the logarithmic velocity distribution was derived for the area near the bed, it appears from measurements that the logarithmic velocity profile is a good approximation for the full depth of the flow due to a simultaneous decrease in shear stress and mixing-length with z.

Values of z_o are found from experiments on smooth and rough boundaries. For smooth boundaries a viscous sublayer exists in which viscous effects predominate. The approximate thickness of this layer is $\delta \approx 10\ \nu/u^*$ (see below) and $z_o \approx 0.01\ \delta \approx 0.1\ \nu/u^*$. For boundaries with uniform roughness Nikuradse has found:

$$z_o \approx 0.03\ k_s$$

in which k_s was the size of the sand grains used as roughness. This k_s is used as a standard roughness for other types of roughness.

Smooth boundary

$z_o \approx 0.01\ \delta$

$U(z) = 5.75\ u^* \log(100z/\delta)$

$\bar{U} = 5.75\ u^* \log(40\ h/\delta)$

Rough boundary

$z_o \approx 0.03\ k_s$

$U(z) = 5.75\ u^* \log(33\ z/k_s)$

$\bar{U} = 5.75\ u^* \log(12\ h/k_s)$

$$\bar{U} = 5.75\ u^* \log\left(\frac{12h}{k_s + 0.3\delta}\right)$$

$$\text{or} \quad \bar{U} = (5.75\sqrt{g}) \cdot \sqrt{h\ I} \cdot \log\left(\frac{12h}{k_s + 0.3\delta}\right)$$

$$\text{or} \quad \boxed{\bar{U} = 18\sqrt{h\ I} \cdot \log\left(\frac{12h}{k_s + 0.3\delta}\right)} \quad \text{(White - Colebrook)} \qquad (5.6)$$

which is the well-known Chézy equation:

$$\boxed{\bar{U} = C\sqrt{h\ I}} \qquad (5.7)$$

A bed is defined as hydraulically smooth for $k_s < 0.1\delta$ or $u^* k_s/\nu < 1$.
hydraulically rough for $k_s > 6\delta$ or $u^* k_s/\nu > 60$.

The value of u^* is related to the velocity distribution by:

$$u^* = \frac{1}{5.75} \cdot \frac{\partial\ U(z)}{\partial\ (\log z)}$$

but this method gives generally inaccurate results.

Viscous sublayer δ

In the viscous sublayer viscosity predominates. The velocity distribution therefore follows from $\tau(z) = \eta\ \partial\ U(z)/\partial z$

$$\tau(z) = \tau_b = \rho g h I = \rho {u^*}^2$$

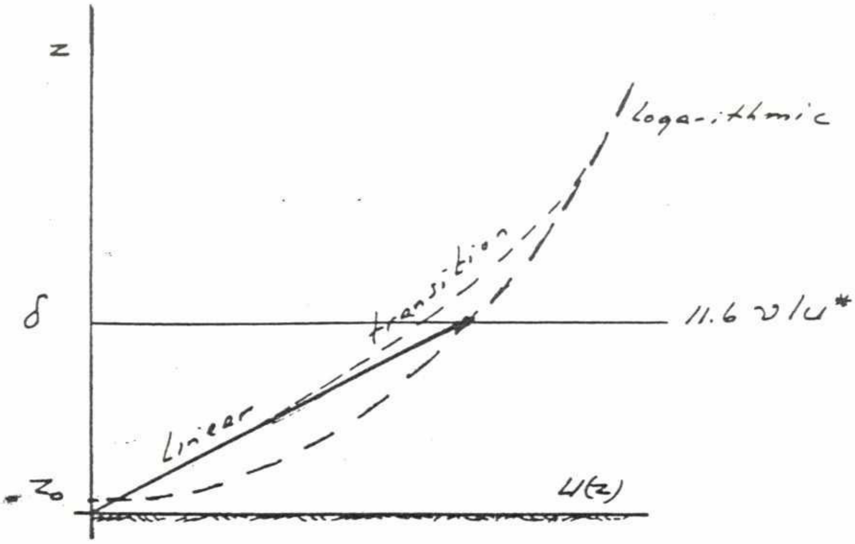
$$\text{or} \quad \frac{U(z)}{u^*} = \frac{u^* z}{\nu}$$

Intersection with the logarithmic velocity distribution gives a "theoretical" value for δ:

$\delta = 11.6\ \nu/u^*$

In fact there is a transition zone from the linear to the logarithmic profile extending from:

$z = (5 \text{ to } 30)\nu/u^*$



5.3. Turbulence

Turbulence is a random fluctuating velocity field which interacts with and derives its energy from the mean flow field. A turbulent velocity field can only be described by statistical quantities such as r.m.s. values, amplitude distribution, correlations and spectra. The amplitudes are generally normally distributed so that the root-mean-square deviation gives a good idea of the fluctuations. $\sigma_u = \overline{(U - \overline{U})^2}$ where U = the instantaneous velocity and $\overline{U}$ the time-averaged value.

A turbulent field has a diffusive character. Gradients of momentum and scalar quantities are rapidly diminished by this diffusive action. The analogy of turbulent motion with the movements of molecules leads to the analogy given by Boussinesq and the introduction of a eddy-viscosity concept for the apparent turbulent shear stress $-\rho \overline{u'w'}$

$$-\rho \overline{u'w'} = \rho \, \varepsilon_m \, \partial U/\partial z$$ (u', w' are velocity fluctuations in horizontal and vertical direction)

so that the total shear stress becomes:

$$\tau = \eta . \frac{\partial U}{\partial z} - \rho \, \overline{u'w'} = \rho \, (\nu + \varepsilon_m) \frac{\partial U}{\partial z}$$

ε_m = eddy viscosity.

The logarithmic velocity distribution:

$$U(z)/u^{*} = \frac{1}{\kappa} \ln (z/z_o)$$

and the linear shear stress distribution:

$$\tau(z) = \tau(b) \; (h - z)/h$$

give the following distribution for $\varepsilon_m(z)$:

$$\varepsilon_m(z) = \kappa\, u^* z(1 - z/h)$$

The average value of $\varepsilon_m(z)$ (averaging over the depth) is therefore:

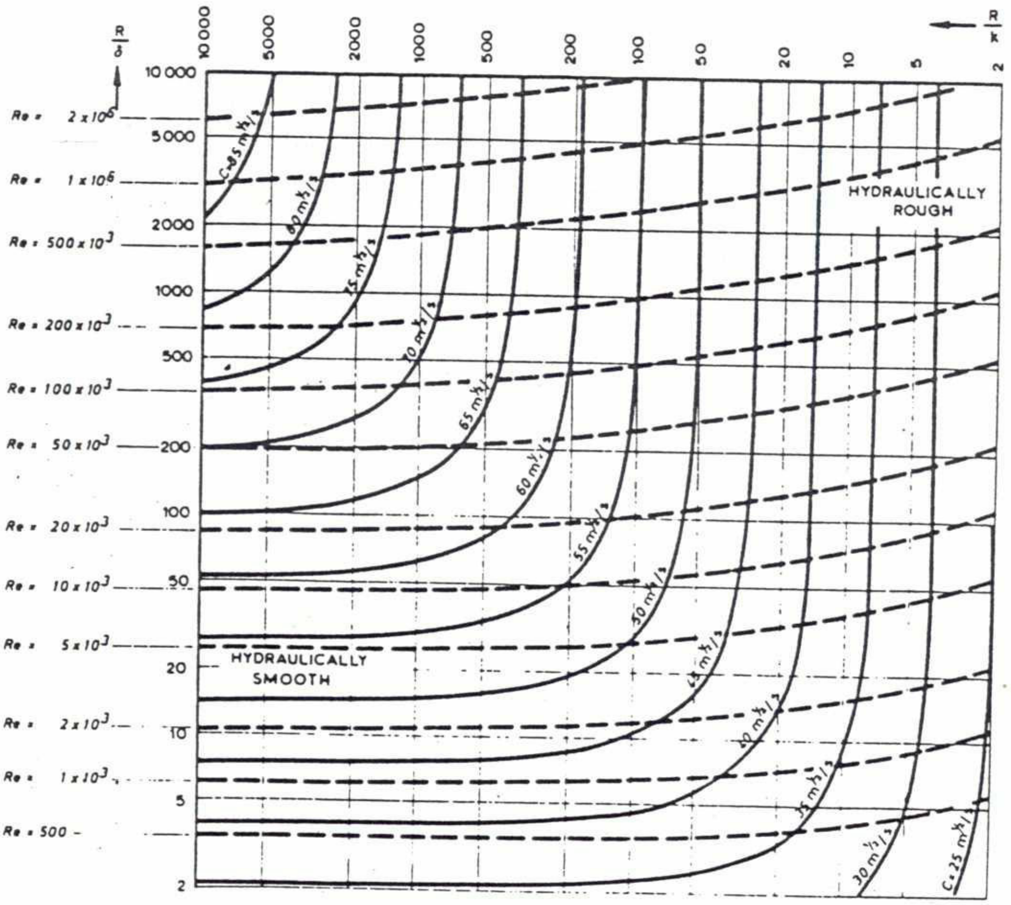
$$\bar{\varepsilon}_m = \frac{1}{6}\, \kappa u^* h.$$

5.4. Resistance laws

The relation for C derived from the White-Colebrook expression :

$$C = 18 \log \left(\frac{12R}{k_s + 0.3\delta}\right) \qquad Re = \frac{\bar{U}R}{\nu} \qquad (5.8)$$

is given in the figure below:



Chézy coefficient for open channels ($m^{\frac{1}{2}}/s$)

Roughness value k_s

For uniform sediment $k_s = D$.

For graded sediment $k_s = D_{65}$ to D_{90}.

For ripples $k_s = (0.5 \text{ to } 1) h_{ripple}$.

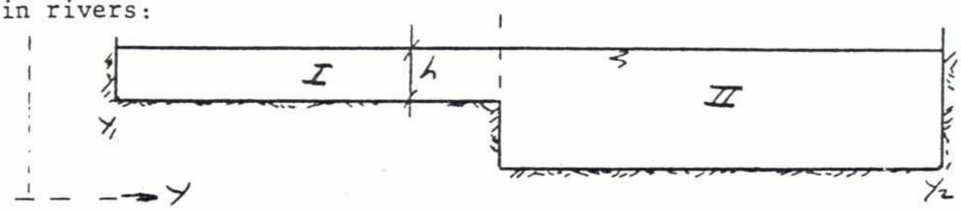
Errors in k_s give the following errors in C:

$\frac{k_{actual}}{k_{estimated}}$	1	2	5	10
$C_{est-act}$	0	5.5	12.5	18

For an arbitrary cross section with surface area A and wetted perimeter P, the equilibrium condition leads to:

$$\overline{\tau_b} = \rho g \frac{A}{P} I = \rho g R I \qquad R = \frac{A}{P} \tag{5.9}$$

in which R is the hydraulic radius. For irregular cross sections also h has to be replaced by R in Eq. 5.8. For a rectangular cross section with width b: $R = bh.(b + 2h)^{-1}$. For composite cross sections as occur in rivers:



it is better to apply (5.7) to the various parts and to add the computed discharges:

$$Q = \int_{y_1}^{y_2} h.\bar{U}dy = \sqrt{I}. \int_{y_1}^{y_2} C.h^{3/2}dy \tag{5.10}$$

A disadvantage of the C value is that it is not dimensionless. A better coefficient is the Darcy-Weisbach friction factor:

$$\lambda = \frac{8g}{C^2} \tag{5.11}$$

(see Par. 5.5 on pipe flow)

The logarithmic expression for C may be approximated for R/k_s = 10 to 1000 by:

$$C = 25 \left(\frac{R}{k_s}\right)^{1/6} \qquad \text{(Strickler)} \tag{5.12}$$

which gives an expression that is equivalent to the Manning equation:

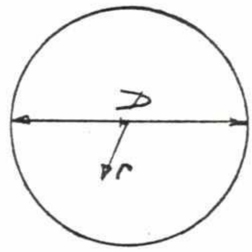
$$\bar{U} = \frac{1.49}{n} . R^{2/3} I^{1/2} \qquad \text{(ft-s units !)} \tag{5.13}$$

$$\bar{U} = \frac{1}{n} R^{2/3} I^{1/2} \qquad \text{(m-s units)} \tag{5.14}$$

For values of Manning's n see Ven Te Chow (1959). Values range from 0.01 for very smooth channels to 0.1 for very rough vegetated channels.

5.5. Flow in circular pipes

From the balance between wall shear stress and forces (pressure and gravity) it follows that:



$$\tau_w = \rho g I \frac{D}{4} \qquad (5.15)$$

I = gradient of piezometric level.

This corresponds to $R = \frac{D}{4}$ $\qquad R = \frac{\pi/4\ D^2}{\pi D} = \frac{D}{4}$

The shear stress distribution is linear:

$$\tau(r) = \tau_w \cdot \frac{2r}{D}$$

For laminar flow the velocity distribution is parabolic:

$$U(r) = \frac{gI}{16\nu}\,(D^2 - 4r^2) \qquad (5.16)$$

Integration gives:

$$\bar{U} = \frac{gI}{32\nu}\,D^2 \qquad \text{(Poiseuille's law)} \qquad (5.17)$$

For turbulent flow the logarithmic distribution (5.5) holds with sufficient accuracy. From this fact a relation between $\bar{U}$ and I can be given.

$$I = \frac{\lambda}{D} \cdot \frac{\bar{U}^2}{2g} \qquad (5.18)$$

with as a useful design relation the White-Colebrook formula based on their and Nikuradse's experiments:

$$\frac{1}{\sqrt{\lambda}} = -2 \log \left(\frac{k_s}{3.71D} + \frac{2.51}{Re\sqrt{\lambda}} \right) \qquad Re = \frac{\bar{U}D}{\nu} \qquad (5.19)$$

The rough wall case corresponds to:

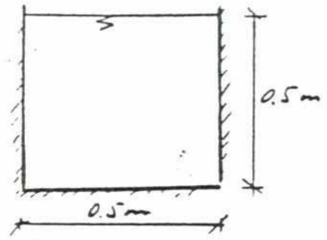
$$C = \sqrt{\frac{8g}{\lambda}} = 17.7 \log \frac{14.8\ R}{k_s} \qquad R = \frac{D}{4} \qquad (5.20)$$

This shows that the use of the hydraulic radius concept gives some minor modifications in the coefficients for various cross sections.

5.6 Problems

$g = 10\ ^{m}/s^2 \quad \nu = 10^{-6}\ m^2/s \quad \rho = 1000\ ^{kg}/m^3$

5.1



A channel has a wall roughness of $k_s = 2$ mm

$B = h = 0{,}5$ m Slope $I = 10^{-3}$

Compute Q.

5.2 A pipe with a diameter of 0.3 m and a length L of 3000 m, and a λ-value of 0.02 has to transport a discharge of $Q = 0.2\ m^3/s$. What is the necessary pressure difference over the pipe section?

5.3 Given: a wide open channel has the following characteristics:

depth $h = 2$ m roughness $k_s = 1$ mm

slope $I = 10^{-5}$

Question: compute $\bar{U}$. (Is the bed rough/smooth/transition?)

same for $k_s = 0.05$ mm and $k_s = 5$ mm.

5.4 Given: wide open channel:

depth $h = 1.2$ m $k_s = 0.5$ mm

discharge/m^1 $q = 0.8\ m^3/s.m$

Question: compute slope I. Is the bed smooth/rough/transition?

5.5 Given: wide open channel:

$k_s = 5$ mm

$I = 2.10^{-5}$

$q = 1.6\ m^3/s.m$

Question: compute depth h.

5.6 Given: measurements in a wide open channel gave the following velocity profile:

$U(z) = 0.148 \log z/z_0$ (U in $^{m}/s$, z in m.)

Questions: 1) compute U^*.

2) compute k_s if the velocity at $z = 0.1$ m was equal to $0.31\ ^{m}/s$.

6. STEADY NON-UNIFORM FLOW

6.1. Introduction

The flow is steady so $\frac{\partial}{\partial t} = 0$

There are two classes of problems:

- gradually varied flow: draw-down and backwater-curves; bedfriction important but assumed equal to that in uniform flow.
- rapidly varied flow: flow through outlets, over dams; friction generally not important.

6.2. Gradually varied flow

6.2.1. General

For small bed slopes i_b and uniform flow Chézy's equation can be applied:

$$\bar{U} = C\sqrt{hI} \qquad I = i_b \tag{6.1}$$

h is used here, because the considerations are limited to wide open channels so $R \simeq h$.

The bed shear stress τ_b may be given therefore as:

$$\tau_b = \rho g h I = \frac{\rho g \bar{U}^2}{C^2} \tag{6.2}$$

For a given discharge q ($m^3/s.m$) the equilibrium depth follows from:

$$q = \bar{U}.h$$

or $$h = h_n = \left[\frac{q^2}{C^2 i_b}\right]^{1/3} \tag{6.3}$$

Of importance is also the critical depth h_c where:

$$Fr = \frac{U}{\sqrt{gh}} = 1$$

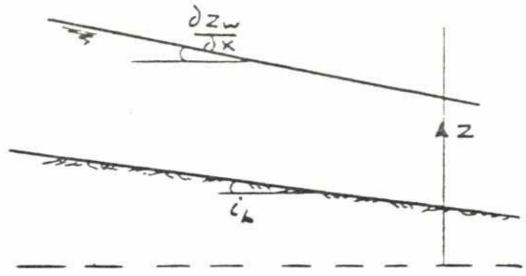
or $$h = h_c = \left[\frac{q^2}{g}\right]^{1/3} \qquad \text{(neglecting } \alpha\text{)} \tag{6.4}$$

Critical uniform flow occurs for $h_n = h_c$ or for a bed slope i_{bc}

$$i_{bc} = \frac{g}{C^2} \tag{6.5}$$

The equation of momentum (4.24) becomes now:

$$\bar{U}\frac{d\bar{U}}{dx} - g\, i_b + g\frac{dh}{dx} + \frac{g\bar{U}^2}{C^2 h} = 0 \qquad (6.6)$$



because $\frac{dz_w}{dx} = -\,i_b + \frac{dh}{dx}$

α' has been taken equal to 1.

The continuity equation was:

$$\frac{dq}{dx} = 0 = \bar{U}\frac{dh}{dx} + h\frac{d\bar{U}}{dx} \qquad (6.7)$$

Elimination of $\frac{d\bar{U}}{dx}$ from (6.6) and (6.7) gives:

$$g\left[1 - \frac{\bar{U}^2}{gh}\right]\frac{dh}{dx} = g\, i_b - g\,\frac{\bar{U}^2}{C^2 h} \qquad (6.8)$$

or $$\frac{dh}{dx} = i_b\,\frac{1 - \dfrac{q^2}{C^2 h^3 i_b}}{1 - \dfrac{\bar{U}^2}{gh}} \qquad (6.9)$$

or with the substitution of h_n and h_c; (6.3) and (6.4):

$$\frac{dh}{dx} = i_b\,\frac{h^3 - h_n{}^3}{h^3 - h_c{}^3} \qquad \text{the equation of } \underline{\text{Bélanger}} \qquad (6.10)$$

(6.10) gives $\frac{dh}{dx} = \infty$ for $h = h_c$, this is not correct, because equation (6.8) shows that $\frac{dh}{dx}$ is undefined in that case.

6.2.2. Draw-down and backwater curves

Eq. (6.10) can be solved if q, C and i_b and a boundary condition are given.

Important: For subcritical flow (Fr < 1) we need a boundary condition at the downstream side, so the computation is in the upstream direction. For supercritical flow (Fr > 1) a boundary condition at the upstream side is needed; the computation is carried out downstream.

The profiles of the water surface depend on:

bed slope:	horizontal slope	$i_b = 0$	type H
	mild slope	$0 < i_b < i_{bc}$	type M
	critical slope	$i_b = i_{bc}$	type C
	steep slope	$i_b > i_{bc}$	type S
	negative slope	$i_b < 0$	type N

and the depth range

zône 1	$h > h_n$	$h > h_c$
zône 2	h between h_c and h_n	
zône 3	$h < h_n$	$h < h_c$

For a classification see Fig. 6.1 (taken from Ven Te Chow)

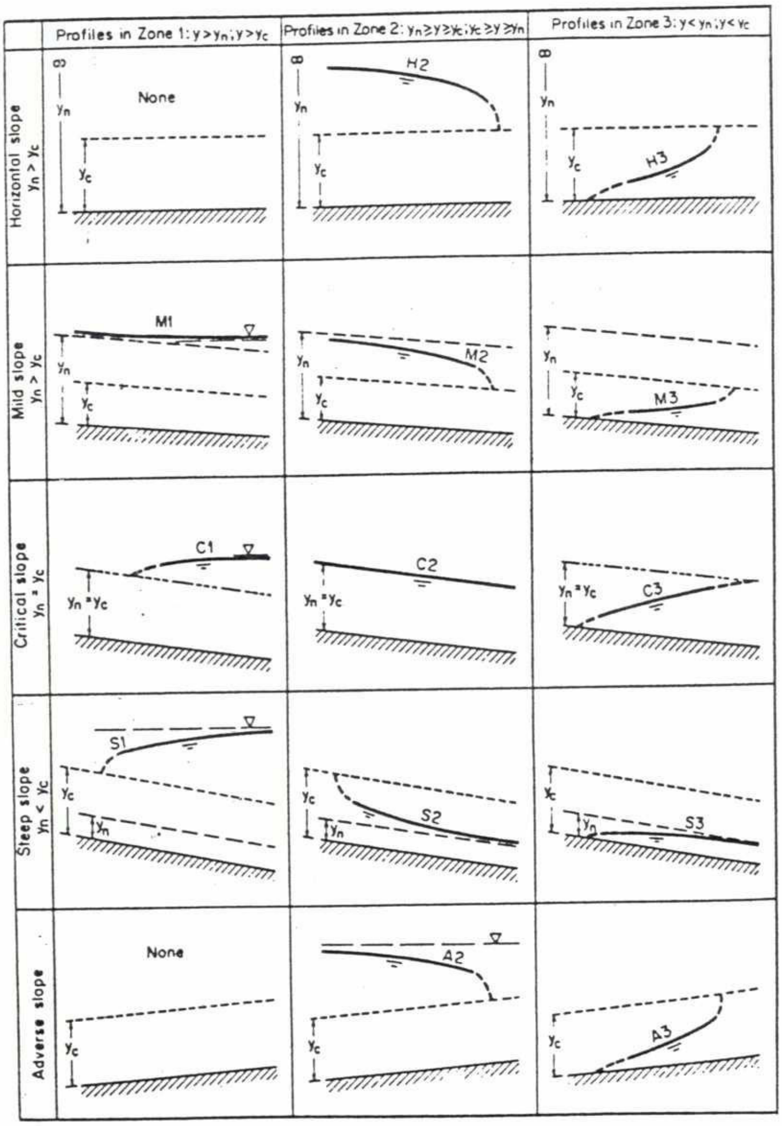


Fig. 6.1. Classification of flow profiles of gradually varied flow.

Some examples are given in Fig. 6.2 (taken from Ven Te Chow)

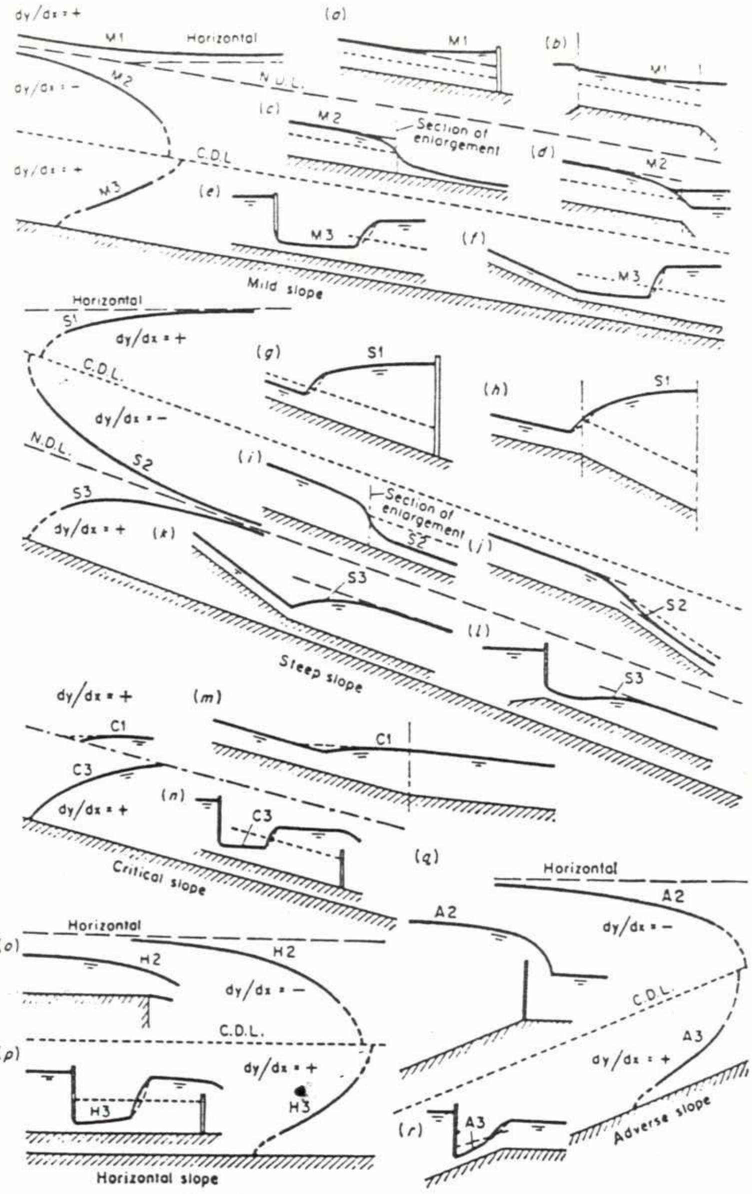


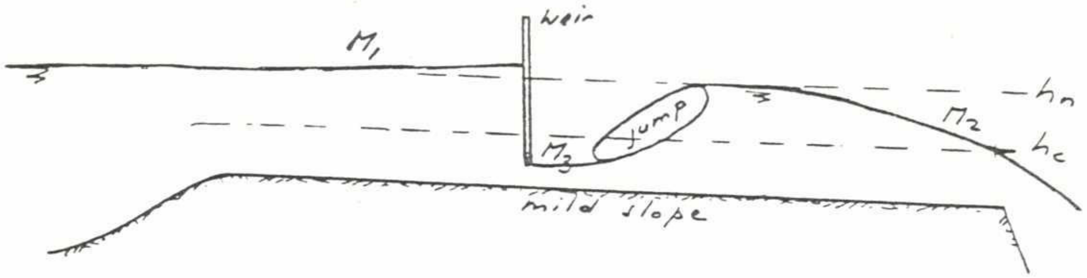
Fig. 6.2. **Examples of flow profiles.**

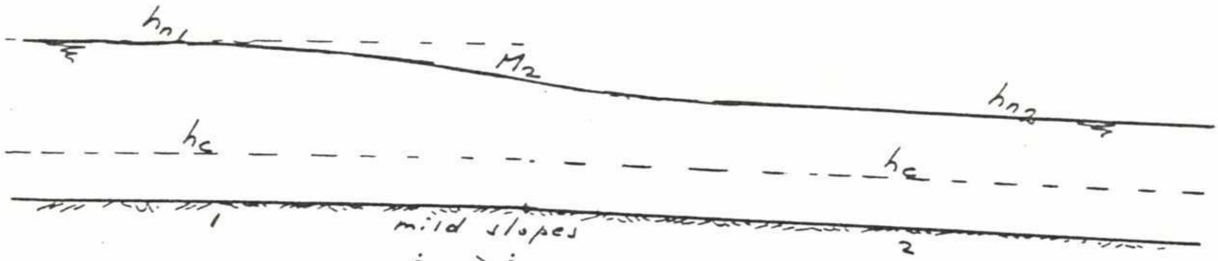
The most frequent curves in river problems are:

M_1: The backwater curve upstream of a dam.
At the dam h is given and $h > h_n$, $h > h_c$, so $\frac{dh}{dx}$ is positive: h decreases in the upstream direction.

M_2: the draw-down curve, for example above a transition from a mild slope to a somewhat greater mild slope.

M_3: supercritical flow downstream of a weir.
The transition of M_3 to M_2 or M_1 gives a hydraulic jump.





$i_{b_2} > i_{b_1}$

6.2.3. Computations

a) Analytical. Function of Bresse.

Eq. (6.10) can be written as: $i_b . \dfrac{dx}{dh} = \dfrac{h^3 - h_c{}^3}{h^3 - h_n{}^3}$ (6.11)

Suppose $h = \eta . h_n$ and $\beta = 1 - (h_c/h_n)^3 = 1 - C^2 i_b/g$ (6.12)

Then $dx = \dfrac{h_n}{i_b} . d\eta - \beta . \dfrac{h_n}{i_b} . \dfrac{d\eta}{1-\eta^3}$ (6.13)

so $x = \dfrac{h_n}{i_b} \left[\eta - \beta \int \dfrac{d\eta}{1-\eta^3} \right] + \text{constant}$ (6.14)

The function of Bresse is defined by:

$$Br(\eta) = \int \frac{d\eta}{1-\eta^3} = \frac{1}{6} \ln \left[\frac{\eta^2+\eta+1}{(\eta-1)^2} \right] - \frac{1}{\sqrt{3}} \text{ arc cotg} \left[\frac{2\eta+1}{\sqrt{3}} \right] \tag{6.15}$$

This function is given in the table below. The constant is omitted in (6.15) because the solution is used in the form:

$$x_2 - x_1 = \frac{h_n}{i_b} \left[(\eta_2 - \eta_1) - \beta \left(Br(\eta_2) - Br(\eta_1) \right) \right] \tag{6.16}$$

η	$Br(\eta)$	η	$Br(\eta)$	η	$Br(\eta)$	η	$Br(\eta)$
0	-0,605	0,90	+0,614	1,002	+1,953	1,18	+0.509
0,10	-0,505	0,91	+0,652	1,005	+1,649	1,20	+0,479
0,20	-0,404	0,92	+0,695	1,010	+1,419	1,25	+0,420
0,30	-0,302	0,93	+0,743	1,02	+1,191	1,30	+0,373
0,40	-0,198	0,94	+0,798	1,03	+1,060	1,35	+0,335
0,50	-0,088	0,95	+0,862	1,04	+0,970	1,40	+0,304
0,60	+0,032	0,96	+0,940	1,05	+0,896	1,50	+0,257
0,65	+0,099	0,97	+1,040	1,06	+0,838	1,60	+0,218
0,70	+0,171	0,98	+1,178	1,07	+0,790	1,70	+0,190
0,75	+0,252	0,990	+1,413	1,08	+0,749	1,80	+0,166
0,80	+0,346	0,995	+1,645	1,09	+0,712	1,90	+0,146
0,82	+0,388	0,998	+1,952	1,10	+0,681	2,00	+0,132
0,84	+0,435	0,999	+2,183	1,12	+0,626	2,50	+0,082
0,86	+0,487	1,000	∞	1,14	+0,580	3,00	+0,055
0,88	+0,546	1,001	+2,184	1,16	+0,541	large	$+1/2\eta^2$

Function of Bresse for a two-dimensional channel.

The table may be used in this way: Compute the value of η_1 in the boundary point 1. Select a value of the depth in point 2 and compute η_2. Then compute $\Delta x = x_2 - x_1$. The stepsize is unlimited as long as the conditions (q, h_n, h_c, i_b) do not change.

b) Numerical method. (Predictor-corrector method)

Use Eq. (6.9) and compute $(\frac{dh}{dx})_1$ in the boundary point 1.

Then compute h_2' in a point 2 at a distance $\Delta x = x_2 - x_1$.
Compute $(\frac{dh}{dx})_2$ with this h_2' in point 2 and take the average value

$$(\frac{dh}{dx})_{1,2} = \frac{1}{2}(\frac{dh}{dx})_1 + \frac{1}{2}(\frac{dh}{dx})_2$$

Compute h_2 with this corrected value of $^{dh}/dx$, and repeat if necessary. This method can also be used if the conditions change along the river (for example if the width of the river and therefore q changes).

The permissible step length depends on the relative changes in h and the required accuracy. In parts with rapidly changing depth, small step sizes have to be chosen.

Many other methods, often in the form of standard computer programs are available in the literature. (Also for pocket calculators).

Example

Given: a river with $q = 1.8\ m^2/s$ $C = 45\ m^{\frac{1}{2}}/s$ $i_b = 10^{-4}$
Downstream level control at x = 0 with water level 0.5 m above h_n. Compute the water levels for $x \neq 0$.

Step 1. General:

$$h_n = \left[\frac{q^2}{C^2 i_b}\right]^{1/3} = 2.52\ m$$

$$h_c = \left[\frac{q^2}{g}\right]^{1/3} = 0.69\ m$$

$h > h_n$, so the flow is of the M_1 type.

$$i << i_c = \frac{g}{C^2}$$

In this case only upstream levels (a back-water curve) can be computed.

$$\beta = 1 - (h_c/h_n)^3 = 0.98 \approx 1.0$$

The boundary condition at x = 0 is $h_o = h_n + 0{,}5\ m = 3.02\ m$
so $\eta_o = 3.02/2.52 = 1.20$.

Step 2. Analytical method (Bresse)

For a given η_i, Br_i, x_i and h_i can be computed

i	η_i	B_{ri}	$-x_i$ (km)	h_i (m)
0	1.20	0.479	0	3.02
1	1.18	0.509	1.26	2.97
2	1.16	0.541	2.57	2.92
3	1.14	0.580	4.06	2.87
4	1.12	0.626	5.72	2.82
5	1.10	0.681	7.61	2.77
6	1.08	0.749	9.83	2.72
7	1.06	0.838	12.57	2.67
8	1.04	0.970	16.40	2.62
9	1.02	1.191	42.13	2.57

Step 3. Numerical method

Take $\eta = h/h_n$. Eq. (6.11) can be written as:

$$\frac{dx}{d\eta} = \frac{h_n}{i_b} \frac{\eta^3-(h_c/h_n)^3}{\eta^3-1} = \frac{h_n}{i_b} \cdot \frac{\eta^3-0.02}{\eta^3-1} = f(\eta) \tag{6.17}$$

The numerical scheme becomes:

$$\frac{x_{i+1} - x_i}{\eta_{i+1} - \eta_i} = \frac{1}{2} \{f(\eta_{i+1}) + f(\eta_i)\}$$

(The corrector step is not necessary because $f(\eta_{i+1})$ can be computed directly).

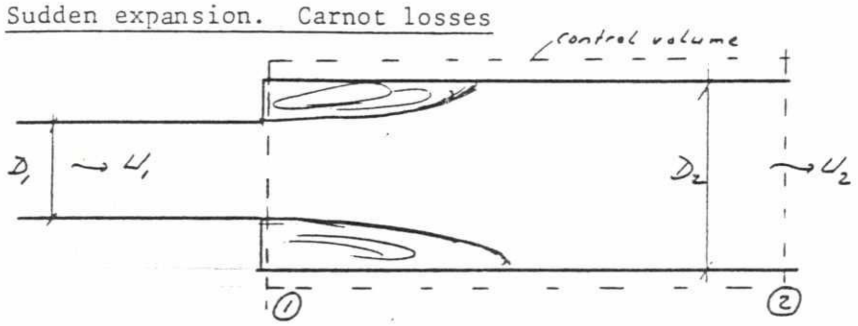
i	h_i m	η_i	$\frac{\eta^3-0.02}{\eta_i^3-1}$	$-(x_{i+1} - x_i)$ km	$-x_i$ km
0	3.02	1.20	2.32		0
				1.30	
1	2.97	1.18	2.50		1.30
				1.32	
2	2.92	1.16	2.73		2.62
				1.45	
3	2.87	1.14	3.01		4.07
				1.62	
4	2.82	1.12	3.40		5.69
				1.85	
5	2.77	1.10	3.94		7.54
				2.20	
6	2.72	1.08	4.75		9.74
				2.75	
7	2.67	1.06	6.11		12.49
				3.76	
8	2.62	1.04	8.80		16.25
				6.52	
9	2.57	1.02	17.0		22.77

For small values of x, the differences between the Bresse and the numerical method are small, but the error increases for large x.

6.3. Rapidly varying flow

In this class of problems, the phenomena are of a limited dimension so wall friction can be neglected in general. In strongly accelerated flow Bernoulli's equation can be used (no energy loss) but in decelerating flows there is a strong influence of Reynolds stresses which cause a large energy dissipation especially in separated flows.

6.3.1. Sudden expansion. Carnot losses



Between section 1 and 2, the flow decelerates which will give some increase in pressure. On the control section acts a pressure difference $p_2 - p_1$ therefore. The hydrostatic pressure can be neglected in this case (hydrostatic and equal in both sections).

Continuity equation: $Q = \bar{U}_1 A_1 = \bar{U}_2 A_2 = \frac{\pi}{4} D_1^{\,2} . \bar{U}_1 = \frac{\pi}{4} D_2^{\,2} \bar{U}_2$

Momentum equation: $\vec{F}dt = dm\vec{U} = \rho Q dt \; (\bar{U}_2 - \bar{U}_1)$

$$-\,(p_2 - p_1)\;\frac{\pi}{4}\,D_2^{\,2} = \rho Q . (\bar{U}_2 - \bar{U}_1)$$

$$\frac{p_2 - p_1}{\rho g} = \frac{\bar{U}_1 \bar{U}_2 - \bar{U}_2^{\,2}}{g} \qquad (6.18)$$

Adding the velocity head on both sections gives:

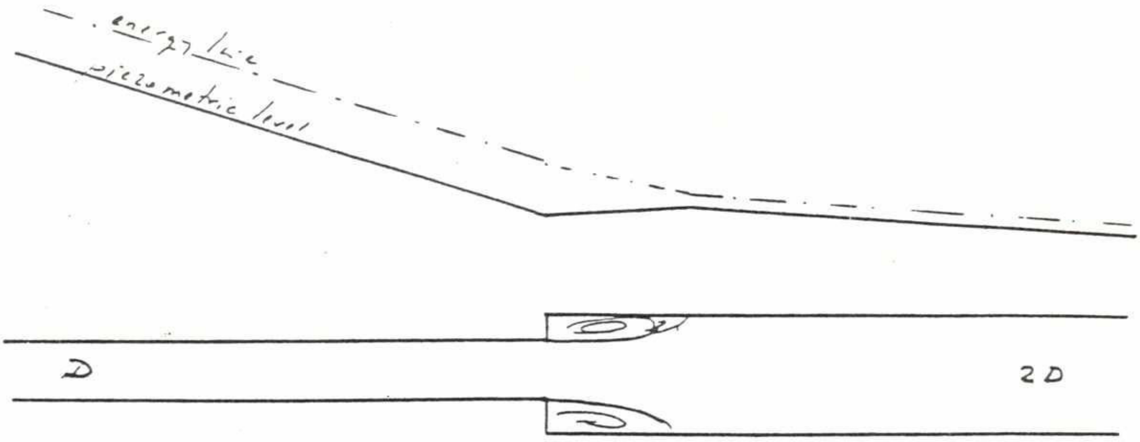
$$\Delta H = H_1 - H_2 = \frac{(\bar{U}_1 - \bar{U}_2)^2}{2g} \qquad (6.19)$$

(the Carnot energy loss for a sudden expansion).
It customary to express the head loss in terms of $\bar{U}_1^{\,2}/2g$. This gives a dimensionless loss coefficient:

$$\xi_{exp} = (1 - \frac{U_2}{U_1})^2 = (1 - \frac{A_1}{A_2})^2 \qquad (6.20)$$

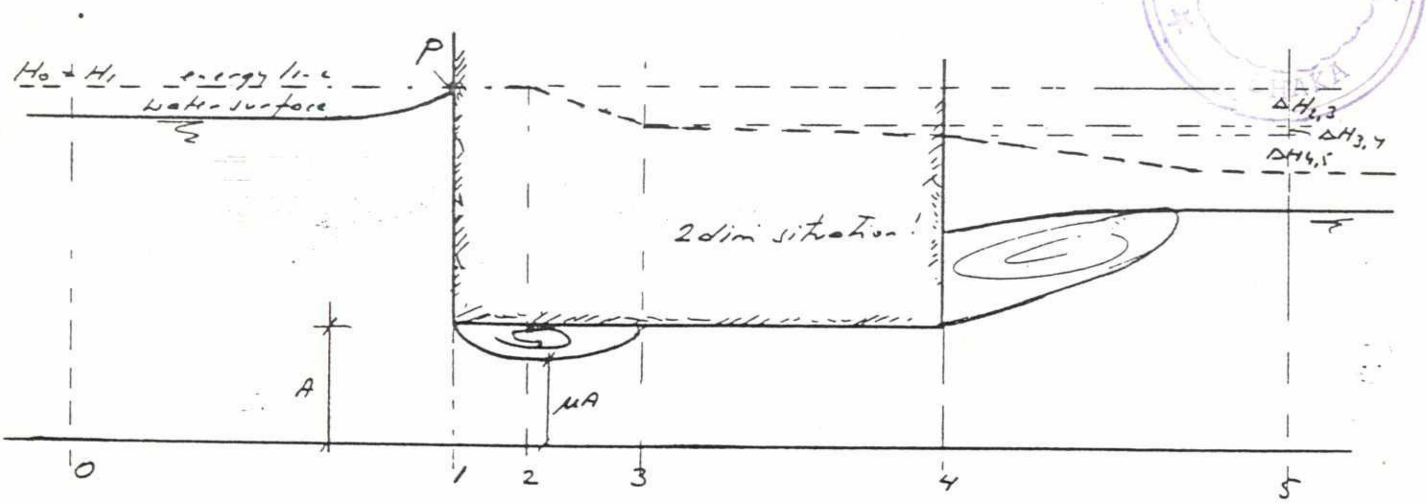
The friction loss in a pipe with a length L and diameter D can also be expressed as a ξ value because:

$$\Delta H = \frac{\lambda L}{D} \cdot \frac{\bar{U}^2}{2g} = \xi_{fr} \cdot \frac{\bar{U}^2}{2g} \qquad \text{or} \qquad \xi_{fr} = \frac{\lambda L}{D} \tag{6.21}$$



The slope of the energy line is much flatter in the second part because the hydraulic gradient varies with D^{-5} ! (prove this).

6.3.2. Entrance losses. Flow through a culvert



The part 0-2 is a zône where strong accelerations occur. Here we can apply Bernoulli's equation, taking into account the contraction in the culvert μA.

$$H_o = H_1 = H_2 = h_1 + \frac{\bar{U}_1^{\,2}}{2g} = h_2 + \frac{U_2^{\,2}}{2g} = h_2 + \frac{q^2}{2g\ (\mu A)^2}$$

h = piezometric level

(Why is there an increase in water level near p ?)

The head loss from 2 to 3 can be computed with the Carnot formula

$$H_2 - H_3 = \frac{(\bar{U}_2 - \bar{U}_3)^2}{2g}$$

Expressing this in the velocity $\bar{U}_3$ in the culvert gives:

$$\xi_{2,3} = (\frac{1}{\mu} - 1)^2 \qquad\qquad \xi = \frac{\Delta H}{\bar{U}_3{}^2/2g}$$

Between 3 and 4 there is a friction loss:

$$H_3 - H_4 = \xi_{fr} \cdot \frac{\bar{U}_3{}^2}{2g}$$

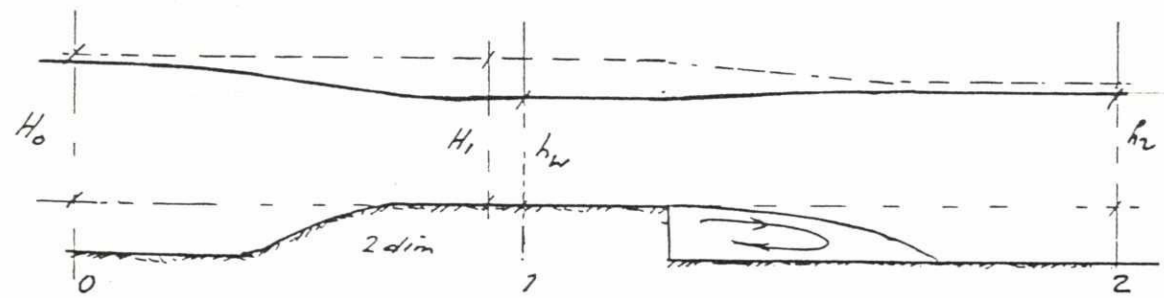
At the exit the loss can be approximated by the Carnot formula

$$\xi_{4,5} = (1 - \frac{A}{h_5})^2$$

The total loss is equal to $H_1 - H_5$. If the discharge and one of the water levels is given (h_o or h_5) then the other levels can be computed.

The entrance loss can be reduced by streamlining the entrance, while the exit loss can be reduced by a gradual increase in profile (diffusor, widening angle $\leq 10^o$).
For information on local losses see for example Lencastre (1969) or Miller (1978).

6.3.3. Broad-crested weir



For low discharges flow is subcritical. Because the weir is long, the stream lines are almost straight and the pressure is hydrostatic. Between section 0 and 1 Bernoulli's equation can be applied or:

$$q = h_w \sqrt{2g(H_1 - h_w)} \qquad\qquad H_1 = H_o \qquad (6.22)$$

Between 1 and 2 there is some water level rise which in certain cases can be computed with the Carnot formula. It is usual however to take into account this effect by introducing a discharge coefficient m, taking h_2 instead of h_w

$$q = m.h_2 \sqrt{2g(H_o - h_2)} \qquad (6.23)$$

An average value for $m \approx 1.1$, for smooth dams $m = 1.3$, for very rough dams $m \approx 0.9$.

If the downstream water level is decreased, keeping H_o constant, then the discharge increases untill the depth at the dam h_w becomes critical:

$$h_w = h_c = \frac{2}{3} H_o$$

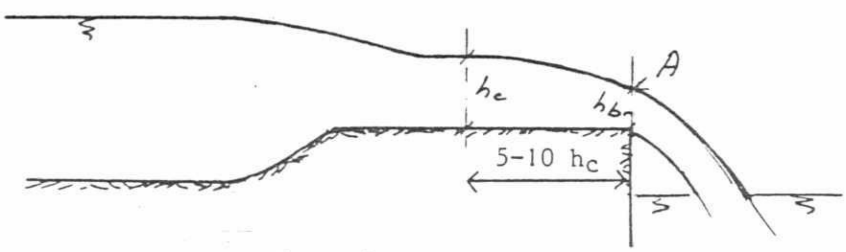
In that case the discharge is given by:

$$q_{max} = m.h_c.\sqrt{2g(H_o - h_c)} = m.\ \frac{2}{3} \sqrt{\frac{2}{3} g}.\ H_o^{3/2} \qquad (6.24)$$

In general $m < 1$ because of friction and entrance losses:

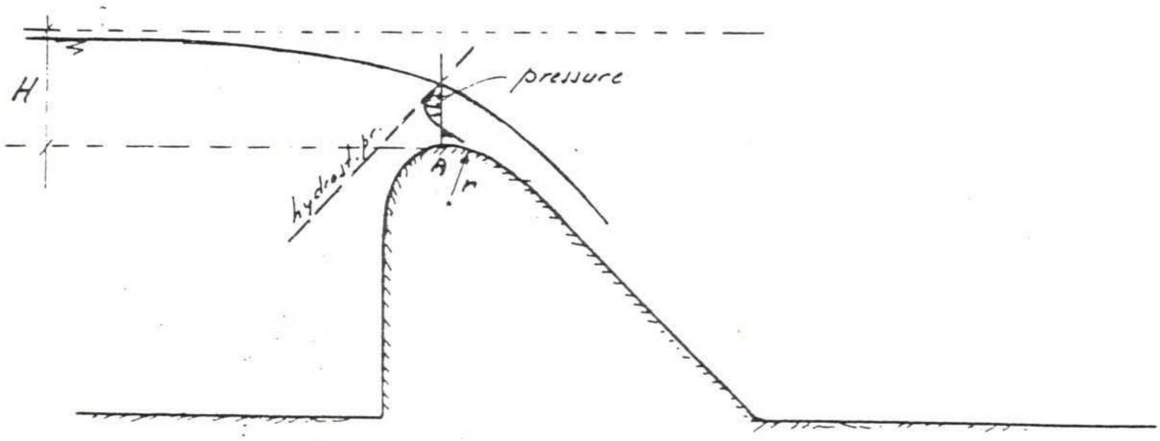
$$0.8 < m < 1.0$$

If the downstream water level is below the crest of the dam, then the water level at the end of the dam is lower than h_c! (Explain: consider the water-pressure at A)



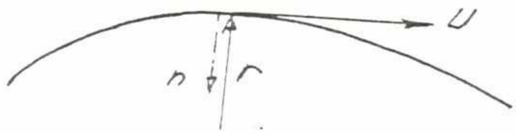
The brink depth h_{br} is equal to $\approx 0.6\ h_c$

6.3.3. Short-crested weir



The flow above the crest of the weir has curved streamlines. This gives a deviation of the hydrostatic pressure distribution.

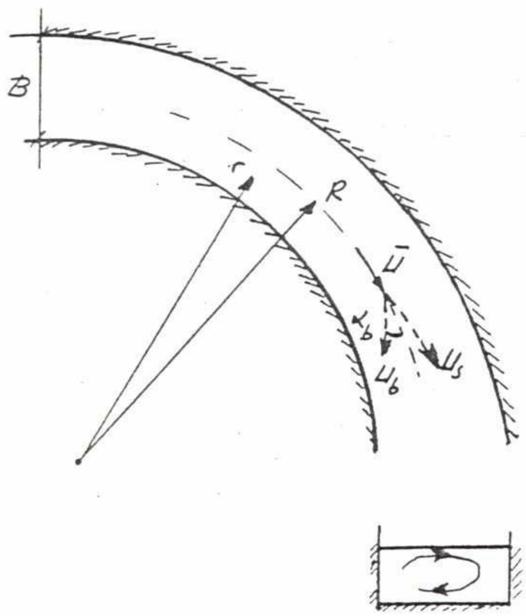
In a flow with a curvature r a pressure gradient:



$$\frac{\partial p}{\partial n} = -\rho \frac{U^2}{r} \quad \text{or} \quad \frac{\partial}{\partial n} \frac{p}{\rho g} = \frac{-U^2}{gr} \tag{6.25}$$

is needed. (Compare the force needed to keep an object in a circular motion. For a more extensive discussion see one of the handbooks.). This means that in curved flow the pressure <u>decreases</u> in the direction of the centre of curvature. The pressure distribution in section A is not hydrostatic therefore, but <u>less</u> than hydrostatic. This means that higher velocities than in the long-crested are possible and that the discharge coefficient m will be larger (maximum value 1.35 for $^r/H$ = 0.7 to 1.0.).

6.4. Flow in river bends



Due to the flow curvature there exists a water surface gradient in the direction of r:

$$\frac{\partial h}{\partial r} = \frac{\bar{U}(r)^2}{gr} \tag{6.26}$$

The velocity varies with depth, but the surface slope depends mainly on the mean velocity $\bar{U}$.

This means that the lateral pressure gradient is too small for surface water particles ($U_s > \bar{U}$) and too large for water particles near the bed ($U_b < \bar{U}$). The surface particles move therefore to the outer bed (increasing r) whereas the bed particles move towards the inner bend (decreasing r). The result is a secondary current. This is important for sediment-transporting rivers because the bed load (particles

(12)

moving over the bed) will move inward. The deviation α_b depends on the dimensions of the bend, the velocity distribution in the vertical and the bed roughness.
As an average:

$$\alpha_b = \frac{10h}{R} \tag{6.27}$$

(see Rozovskii 1957)

The water surface slope is of the order:

$$\bar{U}^2/gR$$

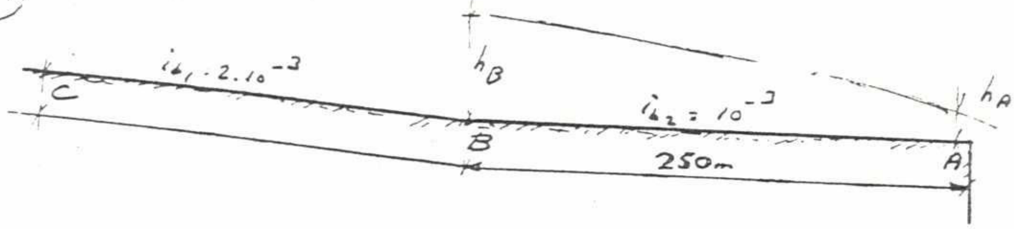
This gives a water level difference of the order:

$$B\bar{U}^2/gR.$$

For a river with $B = 100$ m $R = 1000$ m $\bar{U} = 1$ m/s this gives a difference of 0.01 m.

6.5 Problems

6.1

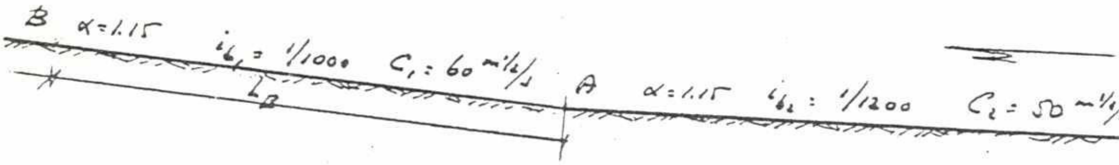


A two-dimensional channel has a change in bed slope at point B, 250 m upstream from a free overfall. $C = 40\ m^{\frac{1}{2}}/s$ $q = 1\ m^3/s.m$ $\alpha = 1$ $g = 10\ m/s^2$.

Questions: a) Compute h_A and h_B assuming critical conditions in A

b) At what distance x upstream of B (point C) is the waterdepth 1.02 times the normal depth?

6.2

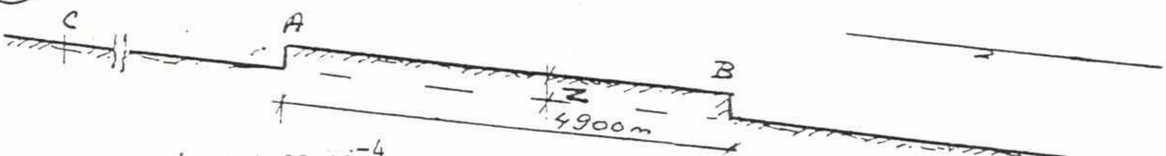


Discharge: $q = 5.36\ m^3/s.m.$ $g = 9.81\ m/s^2$

a) At which point B is the waterdepth equal to 1.05 times the normal depth? (compute L_B). How large is the depth at point A?

b) What type(s) of backwater curve(s) do you expect? Sketch the water surface profile.

6.3



$i_b = 1,25.10^{-4}$

$C = 60\ m^{\frac{1}{2}}/s$ $\alpha = 1.09$

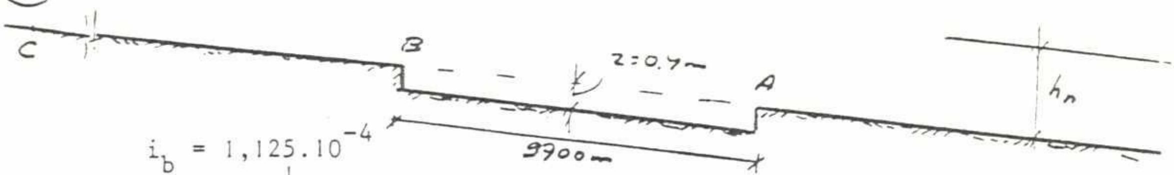
$Q = 243\ m^3/s$ $g = 9.81\ m/s^2$

$B = 150$ m.

In this channel, the bed is raised with $z = 0.36$ m. Compute the water depths at A and B. At which point C is the water depth 1.05 m times the normal depth.

Sketch the water surface; what type(s) of backwater curve(s) do you expect?

(6.4)



$i_b = 1,125.10^{-4}$

$C = 60\ m^{\frac{1}{2}}/s$

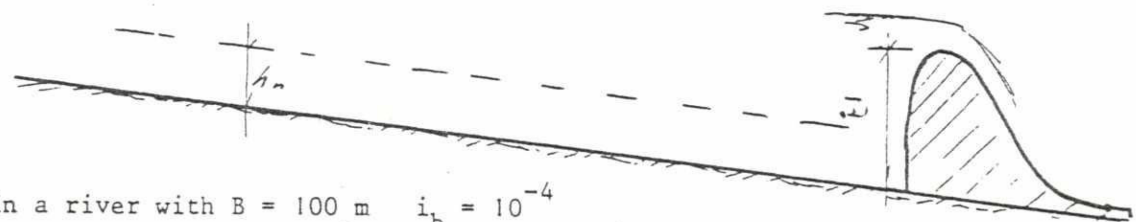
$\alpha = 1.09 \qquad q = 1.8\ m^3/s.m.$

$g = 9.81\ m/s^2$

Compute h_A and h_B if the channel bed between A and B is lowered with 0.4 m.

For what distance BC is the depth in point C equal to 0.95 times the normal water depth?

(6.5)

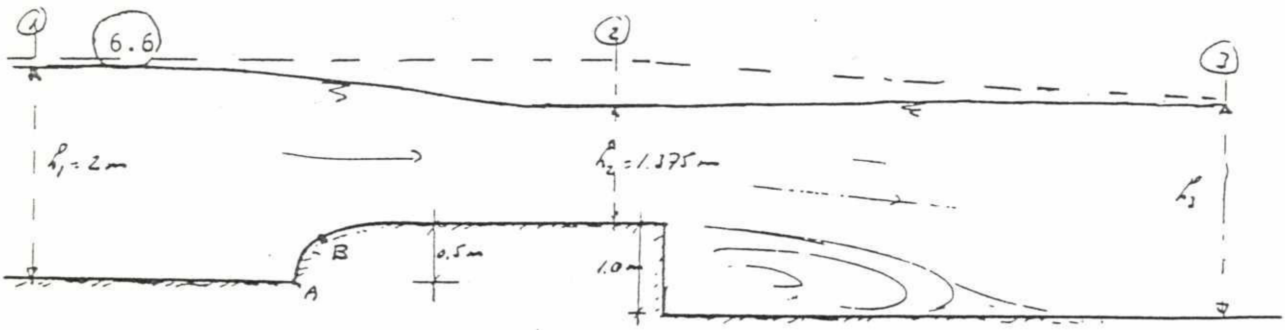


In a river with $B = 100$ m $\quad i_b = 10^{-4}$

$q = 2\ m^3/s.m' \qquad C = 50\ m^{\frac{1}{2}}/s$

a dam is constructed with B = 40 m discharge coeff. m = 1.2 D = 8 m

Compute the water depth at the dam and the distance upstream where the water depth is equal to 1.1 times the normal depth.



Given: a two-dimensional flow with $h_1 = 2$ m and $h_2 = 1.375$ m.

Questions: a) How large is the discharge per m' width (q) assuming that there is no energy loss between sections 1 and 2 ?

b) Compute h_3

c) How large is the energy loss between sections 2 and 3 ?

d) How large is the pressure (in N/m^2) in point A?

e) Is the pressure in B equal to the hydrostatic pressure or smaller/greater than that value?

7. UNSTEADY FLOW

7.1. General

The equations of continuity and motion for flow in a river were:
(taking $\alpha' = 1.0$ $\quad \partial z_w/\partial_x = \partial h/\partial x - i_b$, $\quad \tau_b = \rho g \bar{U}^2/C^2$, see par. 4.3)

2 dimensional $\quad R = h$

Continuity $\quad \dfrac{\partial h}{\partial t} + \dfrac{\partial q}{\partial x} = 0$ (7.1)

Motion $\quad \dfrac{\partial \bar{U}}{\partial t} + \bar{U}\dfrac{\partial \bar{U}}{\partial x} + g\dfrac{\partial h}{\partial x} - g i_b + \dfrac{g\bar{U}^2}{C^2 h} = 0$ (7.2)

3 dimensional

Continuity $\quad B\dfrac{\partial h}{\partial t} + \dfrac{\partial Q}{\partial x} = 0$ (7.3)

Motion $\quad \dfrac{\partial Q}{\partial t} - \dfrac{2QB}{A_c} \cdot \dfrac{\partial h}{\partial t} + gA_c\left(1 - \dfrac{Q^2 B_c}{gA_c{}^3}\right)\dfrac{\partial h}{\partial x} - gA_c i_b + \dfrac{gQ^2}{C^2 R A_c} = 0$ (7.4)

Different approximations are possible depending on the phenomena concerned.

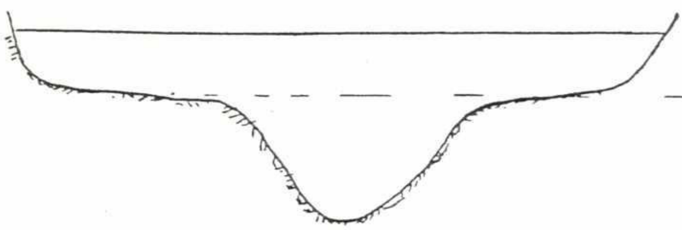
7.2. Steady flow. Rating curve

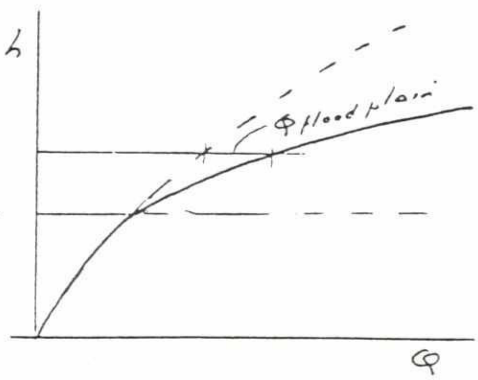
The rating curve gives the relation between Q and h in uniform steady flow. For a 2-dimensional river:

$Q \sim h^{3/2}$ (Chézy. C constant)

$Q \sim h^{5/3}$ (Manning. n constant).

For a river with flood plains, the rating curve will be flatter (a smaller increase in h).





7.3. Flood waves

7.3.1. Kinematic wave

For very "slow" waves, with relatively small increases in discharge and depth it may be assumed that the flow is quasi-steady at any time. This reduces the equation of motion to:

$$Q = B_c C\ i_b^{1/2}\ h^{3/2} \tag{7.5}$$

$(A_c = B_c \cdot h. \quad R \simeq h)$

Introducing (7.5) in (7.3) gives:

$$B\frac{\partial h}{\partial t} + \frac{dQ}{dh} \cdot \frac{\partial h}{\partial x} = 0$$

or: $$\frac{\partial h}{\partial t} + \frac{3}{2}\frac{B_c}{B}\bar{U}\frac{\partial h}{\partial x} = 0 \tag{7.6}$$

This equation represents an undamped wave (the kinematic wave) which has a celerity $c = B^{-1}.\ dQ/dh$ (in general $c = dQ/dA$) or:

$$c = \frac{3}{2}\frac{B_c}{B} . \bar{U} \quad (c = \frac{3}{2}\bar{U} \text{ in the 2 dim. case})) \tag{7.7}$$

(for the Manning equation the coefficient is $^5/_3$)

7.3.2. Diffusive wave

The kinematic wave approach is only valid under limited conditions. (see Grijsen and Vreugdenhil 1976). A better approximation is obtained by leaving the term $\partial h/\partial x$ in the equation, still neglecting the time-dependent terms.

This leads to (2 dimensional):

$$\frac{\partial h}{\partial x} = i_b - \frac{\bar{U}^2}{C^2 h} = i_b - \frac{q^2}{C^2 h^3} \tag{7.8}$$

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = 0 \tag{7.9}$$

q can be eliminated by differentiating (7.8) for constant i_b and C giving:

$$\frac{\partial^2 h}{\partial x^2} = -\frac{2q}{C^2 h^3} \cdot \frac{\partial q}{\partial x} + \frac{3q^2}{C^2 h^4}\frac{\partial h}{\partial x} \tag{7.10}$$

Elimination of $\frac{\partial q}{\partial x}$ gives:

$$\frac{\partial h}{\partial t} - \frac{C^2h^3}{2q} \cdot \frac{\partial^2 h}{\partial x^2} + \frac{3}{2}\,\bar{U}\,\frac{\partial h}{\partial x} = 0$$

or $$\frac{\partial h}{\partial t} - k\,\frac{\partial^2 h}{\partial x^2} + c\,\frac{\partial h}{\partial x} = 0 \qquad (7.11)$$

with $k = \frac{C^2h^3}{2q}$ and $c = \frac{dq}{dh} = \frac{3}{2}\,\bar{U}$

For small variations k and c are constant (linear approximation), (7.11) has the character of a diffusion equation, with a damped wave with celerity $c = {}^3/2\ \bar{U}$ as solution.
Comparison of the diffusive wave with the kinematic wave shows that for the first type:

$$q = Ch^{3/2}\,(i_b - {}^{\partial h}/{\partial x})^{1/2} \qquad (7.12)$$

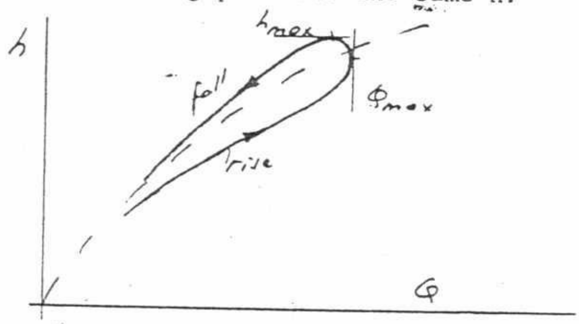
Following the wave it may be assumed (approximation !) that:

$$\frac{\partial h}{\partial x} = -\,\frac{1}{c}\cdot\frac{\partial h}{\partial t} \qquad (7.13)$$

which gives Jones' formula (Henderson 1963):

$$Q = Q_{steady}\ \left(1 + \frac{1}{i_b c}\cdot\frac{\partial h}{\partial t}\right)^{\frac{1}{2}} \qquad (7.14)$$

(7.13) is not valid near the peak of the wave, but is a reasonable approximation. The relation between q and h depends on ${}^{\partial h}/{\partial x}$!
The discharge for a given h is larger in the rising part of the wave than in the falling part for the same h.



For "fast" waves the inertia effects are also of importance, so the full equations have to be considered, which requires a numerical solution in general (for a more detailed review see Jansen 1979 and Cunge et al. (1980)).

7.4. Dynamic waves. Translatory waves

In the dynamic approach for "fast" waves, the inertia effects are dominant. Friction effects can be neglected as a first approximation.

Assumptions:
- friction can be neglected: $g\bar{U}^2/C^2h \simeq 0$
- small Froude number $\bar{U}.\partial\bar{U}/\partial x \simeq 0$
- wave height η small $\eta = h - h_o \ll h_o$ or $\bar{U}.\partial h/\partial x \simeq 0$
- horizontal bed $i_b = 0$

The result is:

motion: $$\frac{\partial\bar{U}}{\partial t} + g\frac{\partial\eta}{\partial x} = 0 \qquad (7.$$

continuity: $$\frac{\partial\eta}{\partial t} + h_o\frac{\partial\bar{U}}{\partial x} = 0 \qquad (7.16)$$

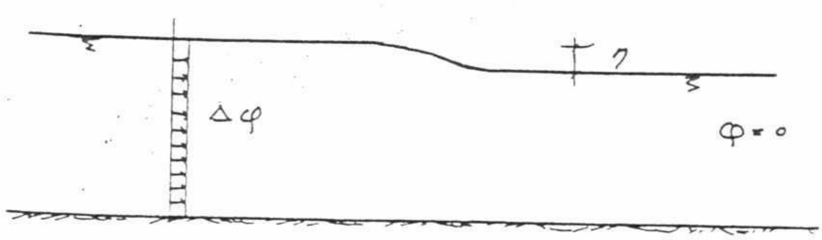
From these equations it follows that:

$$\frac{\partial^2\eta}{\partial t^2} - g\,h_o\frac{\partial^2\eta}{\partial x^2} = 0 \quad \text{and} \quad \frac{\partial^2\bar{U}}{\partial t^2} - gh_o\frac{\partial^2\bar{U}}{\partial x^2} = 0 \qquad (7.17)$$

or $$\frac{\partial^2\eta}{\partial t^2} - c^2\frac{\partial^2\eta}{\partial x^2} = 0 \qquad \text{with } c = \pm\sqrt{gh_o}$$

The solutions in this case (no initial water motion) are waves with celerity $c = \pm\sqrt{gh_o}$. In situations with an initial velocity U_o the celerity becomes $c = U_o \pm \sqrt{gh_o}$.
Examples are waves generated by the discharge from shiplocks or the unsteady releases from power plants. The relation between η and ΔQ is given by:

$$\eta = \frac{\Delta Q}{Bc} \qquad (7.18)$$



The celerity in the three-dimensional case is given by:

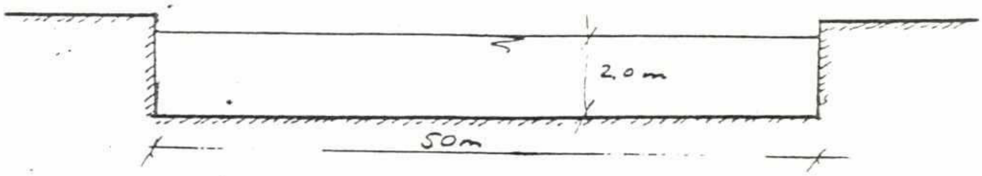
$$c = \pm\sqrt{\frac{gA_c}{B}} \qquad (7.19)$$

For larger values of η, the wave becomes steeper and can even form a breaking wave (tidal bore).
The propagation of a disturbance (with velocity $c = U_0 \pm \sqrt{gh}$) is much faster than that of a flood wave ($c \approx 3/2\ U_0$). This not in contradiction: Adding an amount of water gives waves which propagate as a dynamic wave but are strongly damped by friction. The added volume (the flood) can not disappear, it is propagated with $c = dq/dh$.

7.5 Problems

$g = 10\ ^{m}/s^{2}$

7.1



A channel has the following characteristics:

width 50 m

depth 2 m

slope 10^{-4}

roughness $k_s = 10^{-2}$ m

Questions:

a) Compute the discharge and average flow velocity for steady flow

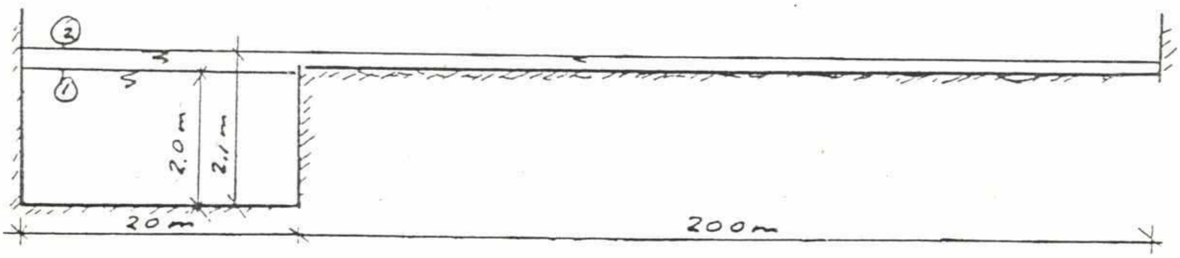
b) Compute the critical depth h_c assuming $\alpha = 1.0$

c) Compute the celerity of a kinematic type flood wave

d) Compute the celerity of a translatory wave

e) Explain the difference between the two wave types.

7.2



A river has a main channel with a width of 20 m and a flood plain with a width of 200 m. The average flow velocity for a depth of 2.0 to 2.1 m is 1.2 $^{m}/s$.

Questions:

a) How large is in situation 1 (no water in the flood plain)

a_1) the celerity of a kinematic type flood wave

a_2) the celerity of a translatory wave ?

b) How will the celerity of the flood wave change if the water level is raised with 0.1 m (situation 2)

c) Explain the difference between flood waves and translatory waves.

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